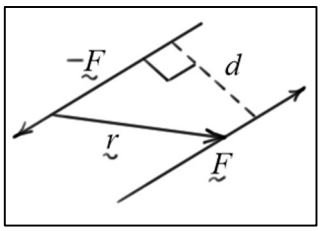
Elementary Statics
Equation Sheet #4: Moments of Couples/Equivalent Force Systems/Rigid Body Equilibrium

Moment of a Couple: $\boxed{M_C = F \cdot d}$ $\boxed{\underset{\sim}{M}_C = \underset{\sim}{r} \times \underset{\sim}{F}}$



Equivalent Force Systems

$$\boxed{\begin{aligned} &\underset{\sim}{F} = \underset{\sim}{F}_R = \sum_i \underset{\sim}{F}_i \qquad \text{(acting at O)} \\ &\underset{\sim}{M}_O = \sum_{\text{forces } (i)} \left(\underset{\sim}{r}_i \times \underset{\sim}{F}_i\right) + \sum_{\text{couples } (i)} \left(\underset{\sim}{M}_C\right)_i \end{aligned}}$$

Centroids of Areas

- Area, A: $\boxed{\overline{x} = \frac{1}{A}\int_A x\, dA}$ $\boxed{\overline{y} = \frac{1}{A}\int_A y\, dA}$
- Composite Area: $\boxed{\overline{x} = \frac{\sum A_i\, \overline{x}_i}{\sum A_i}}$ $\boxed{\overline{y} = \frac{\sum A_i\, \overline{y}_i}{\sum A_i}}$

Equivalent Force Systems for Distributed Loads

$$\boxed{F_R = \sum F = \int_0^L w(x)\, dx} \qquad \boxed{\overline{x} = \frac{1}{F_R}\int_0^L x\, w(x)\, dx}$$

Rigid Body Equilibrium

Vector Equations: $\boxed{\begin{aligned} &\underset{\sim}{F}_R = \sum_i \underset{\sim}{F}_i = \underset{\sim}{0} \\ &\underset{\sim}{M}_P = \sum_i \left(\underset{\sim}{p}_i \times \underset{\sim}{F}_i\right) = \underset{\sim}{0} \end{aligned}}$

Scalar Equations in 2D: $\boxed{\begin{aligned} &\sum F_x = 0 \\ &\sum F_y = 0 \\ &\sum M_P = 0 \end{aligned}}$ or $\boxed{\begin{aligned} &\sum F_x = 0 \quad \text{-or-} \quad \sum F_y = 0 \\ &\sum M_P = 0 \\ &\sum M_Q = 0 \end{aligned}}$ or $\boxed{\begin{aligned} &\sum M_P = 0 \\ &\sum M_Q = 0 \\ &\sum M_R = 0 \end{aligned}}$ (P, Q, R not colinear)

Scalar Equations in 3D: $\boxed{\begin{aligned} &\sum F_x = 0 \\ &\sum F_y = 0 \\ &\sum F_z = 0 \end{aligned}}$ and $\boxed{\begin{aligned} &\sum \left(M_P\right)_x = 0 \\ &\sum \left(M_P\right)_y = 0 \\ &\sum \left(M_P\right)_z = 0 \end{aligned}}$