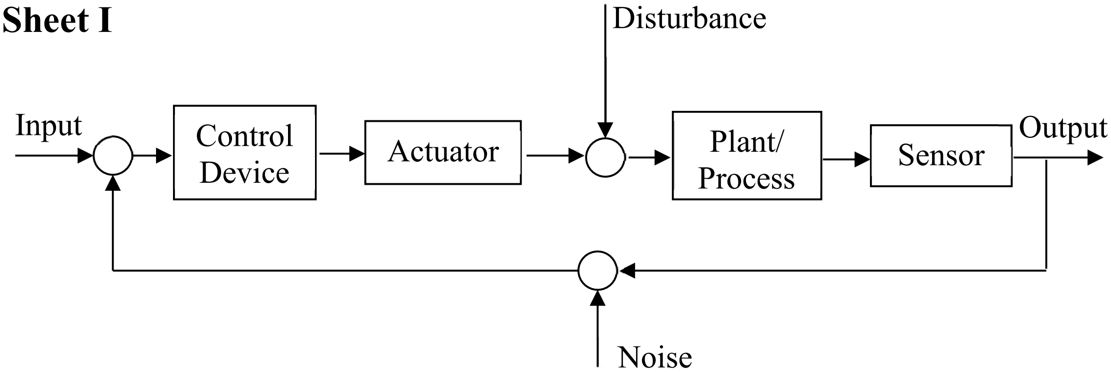
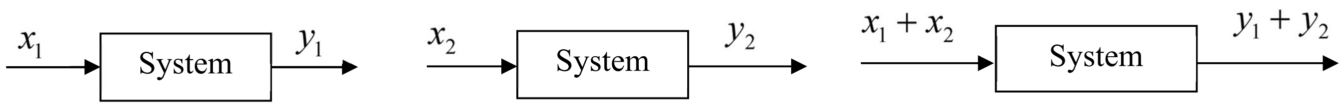
Introductory Control Systems
Equation Sheet I

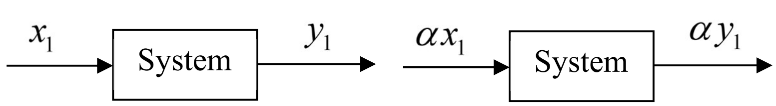


General Quadratic Equation: $\boxed{s^2 + 2\zeta\omega_n s + \omega_n^2 = 0}$ **Roots**: $\boxed{s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}}$

Principle of Superposition:



Principle of Homogeneity: (α is a scalar)



Linear Model: $\boxed{\Delta f = m\,\Delta x}$ and $\boxed{m = \left.\dfrac{df}{dx}\right|_{x=x_{eq}}}$

Properties of the Laplace Transform and Inverse Laplace Transforms:

1. $\mathscr{L}(kf(t)) = k\mathscr{L}(f(t)) = kF(s)$ and $\mathscr{L}^{-1}(kF(s)) = kf(t)$
2. $\mathscr{L}\big(f_1(t) \pm f_2(t)\big) = F_1(s) \pm F_2(s)$ and $\mathscr{L}^{-1}(F_1(s) \pm F_2(s)) = f_1(t) \pm f_2(t)$
3. $\mathscr{L}\left(\dfrac{df}{dt}\right) = sF(s) - f(0)$ and $\mathscr{L}\left(\dfrac{d^2 f}{dt^2}\right) = s^2F(s) - s\,f(0) - \dfrac{df}{dt}(0)$
4. Final Value Theorem: $\lim\limits_{t\to\infty}\big(f(t)\big) = \lim\limits_{s\to 0}\big(s\,F(s)\big)$ (if the time limit exists)

General Partial Fraction Expansion:

$$X(s) = \frac{K_1}{(s+s_1)} + \frac{K_2}{(s+s_2)} + \cdots + \frac{K_N}{(s+s_N)} +$$
$$\frac{A_1 s + B_1}{(s^2 + 2\zeta\omega_n s + \omega_n^2)_1} + \frac{A_2 s + B_2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)_2} + \cdots + \frac{A_M s + B_M}{(s^2 + 2\zeta\omega_n s + \omega_n^2)_M}$$

$K_i = \big[(s+s_i)X(s)\big]_{s=-s_i}$ and the A and B coefficients are found by clearing fractions.