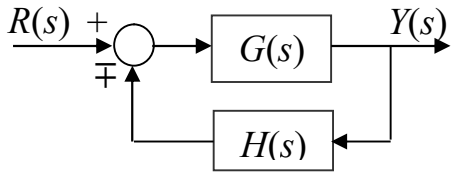
Introductory Control Systems: Equation Sheet II



Sensitivity

$$S_{\alpha}^{T} = \frac{\alpha}{T}\left(\frac{\partial T}{\partial \alpha}\right)$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\left(\frac{du}{dx}\right) - u\left(\frac{dv}{dx}\right)}{v^2}$$

Closed Loop Transfer Function

$$\frac{G}{1 \pm GH} = \frac{(N_G / D_G)}{1 \pm (N_G N_H / D_G D_H)} = \frac{N_G D_H}{D_G D_H \pm N_G N_H}$$

Overshoot for Second-Order Systems

$$T(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$ **Case 1**

$$T(s) = \frac{K(s+a)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$ **Case 2**

System Type and Steady-State Error

$$GH(s) = \frac{N(s)}{s^n D(s)}$$

Type (n)	Steady State System Error		
	Step	*Ramp*	*Parabolic*
0	finite	∞	∞
1	0	finite	∞
2	0	0	finite
3	0	0	0

Time Constants/Settling Times

First Order System

$$\tau = 1/a$$
$$T_s = 4\tau = 4/a$$

Second Order System

$$\tau = 1/\zeta\omega_n$$
$$T_s = 4\tau = 4/\zeta\omega_n$$

Coefficients for ITAE Optimal Performance (Step Input)

$$T(s) = \frac{b_0}{s^n + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0}$$

Order	Form of Denominator
2	$s^2 + 1.4\omega_n s + \omega_n^2$
3	$s^3 + 1.75\omega_n s^2 + 2.15\omega_n^2 s + \omega_n^3$
4	$s^4 + 2.1\omega_n s^3 + 3.4\omega_n^2 s^2 + 2.7\omega_n^3 s + \omega_n^4$
5	$s^5 + 2.8\omega_n s^4 + 5.0\omega_n^2 s^3 + 5.5\omega_n^3 s^2 + 3.4\omega_n^4 s + \omega_n^5$
6	$s^6 + 3.25\omega_n s^5 + 6.60\omega_n^2 s^4 + 8.6\omega_n^3 s^3 + 7.45\omega_n^4 s^2 + 3.95\omega_n^5 s + \omega_n^6$

Complex Roots

$$-\sigma \pm j\omega = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$
$$\omega_n = \sqrt{\sigma^2 + \omega^2}$$
$$\zeta = \sigma / \omega_n$$

Coefficients for ITAE Optimal Performance (Ramp Input)

$$T(s) = \frac{b_1 s + b_0}{s^n + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0}$$

Order	Form of Denominator
2	$s^2 + 3.2\omega_n s + \omega_n^2$
3	$s^3 + 1.75\omega_n s^2 + 3.25\omega_n^2 s + \omega_n^3$
4	$s^4 + 2.41\omega_n s^3 + 4.93\omega_n^2 s^2 + 5.14\omega_n^3 s + \omega_n^4$
5	$s^5 + 2.19\omega_n s^4 + 6.5\omega_n^2 s^3 + 6.3\omega_n^3 s^2 + 5.24\omega_n^4 s + \omega_n^5$