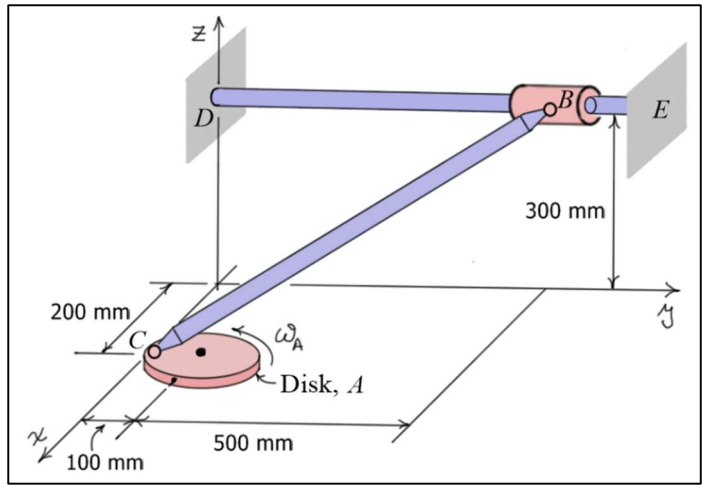
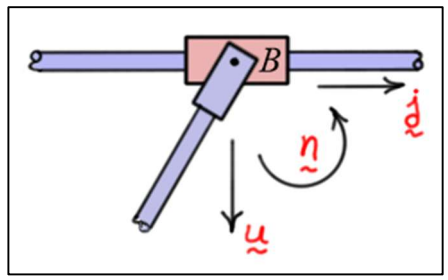
Intermediate Dynamics
Exercises #3

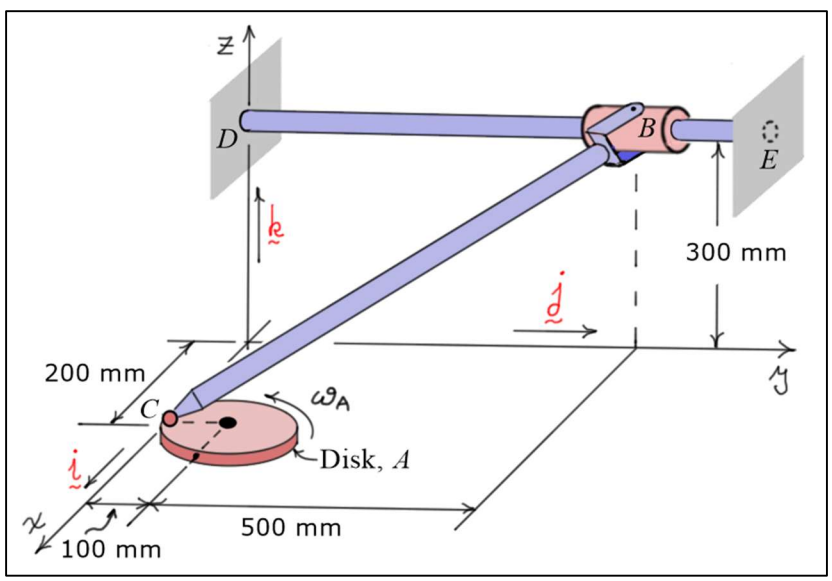
1) The figure shows a three-dimensional slider crank mechanism. The (x, y, z) axes shown represent a fixed reference frame R. At the instant shown, disk A has angular velocity ${}^{R}\underset{\sim}{\omega}_A = 10\underset{\sim}{k}$ (rad/sec).



a) If BC is attached to both the disk and the collar with ball-and-socket joints, find ${}^{R}\underset{\sim}{\omega}_{BC}$ and ${}^{R}\underset{\sim}{v}_B$. In this case, assume that ${}^{R}\underset{\sim}{\omega}_{BC}$ is ***perpendicular*** to BC. (Hibbeler, 1995)

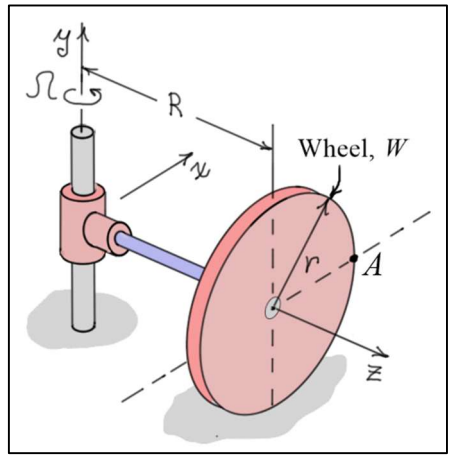
b) If the ball-and-socket joint at the collar is replaced with the ***two-axis joint*** shown at the right, find ${}^{R}\underset{\sim}{\omega}_{BC}$ and ${}^{R}\underset{\sim}{v}_B$. In this case, the rod BC rotates relative to collar C about the axis defined by the unit vector $\underset{\sim}{n}$ which is perpendicular to the plane BCD, and the collar translates and rotates about the $\underset{\sim}{j}$ direction. In this case, ${}^{R}\underset{\sim}{\omega}_{BC}$ is ***perpendicular*** to the vector $\underset{\sim}{u} = \underset{\sim}{j} \times \underset{\sim}{n}$. (Hibbeler, 1995)



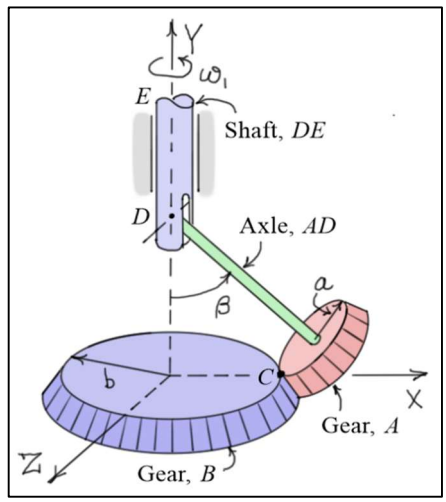


Slider-Crank with Joint of Part (b)

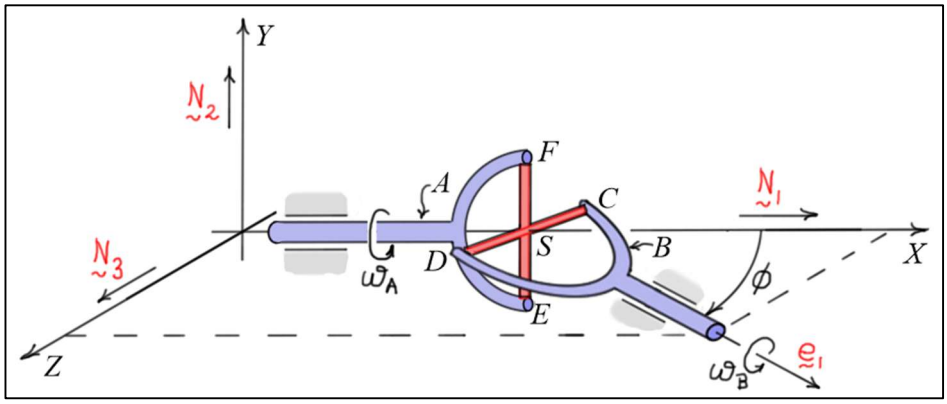
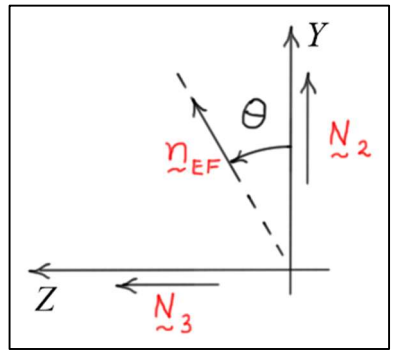
2) The system shown consists of a vertical column, a horizontal axle, and a wheel of radius r. The horizontal arm rotates at a ***constant*** rate Ω, and the wheel (W) rolls without slipping in a circular arc. Find $\underset{\sim}{\omega}_W$ and $\underset{\sim}{\alpha}_W$ the angular velocity and angular acceleration of the wheel relative to a fixed frame, and find $\underset{\sim}{v}_A$ and $\underset{\sim}{a}_A$ the velocity and acceleration of point A. (Meriam and Kraige, 1992)



3) In the system shown, beveled gear A rolls on beveled gear B. As it rolls on B it spins about the axle AD which is pinned to the vertical shaft DE. If DE rotates at a ***constant*** angular velocity ω_1 (rad/sec), find ${}^{AD}\underset{\sim}{\omega}_A$ the angular velocity of gear A relative to the axle AD, ${}^{R}\underset{\sim}{\omega}_A$ the angular velocity of gear A, ${}^{R}\underset{\sim}{\alpha}_A$ the angular acceleration of gear A, and $\underset{\sim}{a}_C$ the acceleration of tooth C of gear A. (Beer & Johnston, 1977)



4) The diagram below depicts a yoke-and-spider ***universal joint***. The joint has three members – the ***input shaft*** A, ***output shaft*** B, and ***spider*** S. Shaft A rotates about the X-axis, and shaft B rotates about the $\underset{\sim}{e}_1$ direction which is in the XZ plane and makes an angle of ϕ with the X-axis. ***At the instant shown***, spider S rotates ***relative*** to shaft A about the direction of spider segment EF (represented by unit vector $\underset{\sim}{n}_{EF} = \underset{\sim}{N}_2$), and it rotates ***relative*** to shaft B about the spider segment CD (represented by unit vector $\underset{\sim}{n}_{CD} = \underset{\sim}{e}_1 \times \underset{\sim}{n}_{EF} = \underset{\sim}{e}_1 \times \underset{\sim}{N}_2$). ***At the instant shown***, find ω_B / ω_A the ratio of the speed of shaft B to the speed of shaft A. Hint: Apply the ***summation rule*** for angular velocities through the joint using the known directions for the angular velocities and relative angular velocities.



5) Referring again to the diagram of the yoke-and-spider ***universal joint*** of problem (4), let input shaft A rotate through an angle θ. The spider segment EF is now at an angle θ relative to the Y-axis, so $\underset{\sim}{n}_{EF} = C_\theta \underset{\sim}{N}_2 + S_\theta \underset{\sim}{N}_3$. As before, shaft B rotates about the $\underset{\sim}{e}_1$ direction, and $\underset{\sim}{n}_{CD} = \underset{\sim}{e}_1 \times \underset{\sim}{n}_{EF}$. Find ω_B / ω_A the ratio of the speed of shaft B to the speed of shaft A as a function of the input shaft angle θ. Hint: Apply the ***summation rule*** for angular velocities ***through the joint*** using the known directions for the angular velocities and relative angular velocities.