Intermediate Dynamics
Exercises #5 Answers

1. (a) $[R] = \begin{bmatrix} C_1C_2 & S_2 & -S_1C_2 \\ S_1S_3 - C_1S_2C_3 & C_2C_3 & C_1S_3 + S_1S_2C_3 \\ S_1C_3 + C_1S_2S_3 & -C_2S_3 & C_1C_3 - S_1S_2S_3 \end{bmatrix}$

   (b) ${}^R\underset{\sim}{\omega}_E = \left(\dot{\theta}_3 + \dot{\theta}_1 S_2\right)\underset{\sim}{e}_1 + \left(\dot{\theta}_2 S_3 + \dot{\theta}_1 C_2 C_3\right)\underset{\sim}{e}_2 + \left(\dot{\theta}_2 C_3 - \dot{\theta}_1 C_2 S_3\right)\underset{\sim}{e}_3$

   (c) $\dot{\theta}_1 = \left(\omega_2 C_3 - \omega_3 S_3\right)/C_2$

   $\dot{\theta}_2 = \omega_2 S_3 + \omega_3 C_3$

   $\dot{\theta}_3 = \omega_1 + \left[S_2\left(\omega_3 S_3 - \omega_2 C_3\right)/C_2\right]$

   d) ${}^R\underset{\sim}{\alpha}_E = \dfrac{{}^Rd}{dt}\left({}^R\underset{\sim}{\omega}_E\right) = \dfrac{{}^Ed}{dt}\left({}^R\underset{\sim}{\omega}_E\right) + \left({}^R\underset{\sim}{\omega}_E \times {}^R\underset{\sim}{\omega}_E\right) = \dfrac{{}^Ed}{dt}\left({}^R\underset{\sim}{\omega}_E\right)$

   $\Rightarrow \boxed{{}^R\underset{\sim}{\alpha}_E = \dot{\omega}_1\,\underset{\sim}{e}_1 + \dot{\omega}_2\,\underset{\sim}{e}_2 + \dot{\omega}_3\,\underset{\sim}{e}_3}$

   Here,

   $\dot{\omega}_1 = \ddot{\theta}_1 S_2 + \dot{\theta}_1\dot{\theta}_2 C_2 + \ddot{\theta}_3$

   $\dot{\omega}_2 = \ddot{\theta}_1 C_2 C_3 - \dot{\theta}_1\dot{\theta}_2 S_2 C_3 - \dot{\theta}_1\dot{\theta}_3 C_2 S_3 + \dot{\theta}_2\dot{\theta}_3 C_3 + \ddot{\theta}_2 S_3$

   $\dot{\omega}_3 = -\ddot{\theta}_1 C_2 S_3 + \dot{\theta}_1\dot{\theta}_2 S_2 S_3 - \dot{\theta}_1\dot{\theta}_3 C_2 C_3 - \dot{\theta}_2\dot{\theta}_3 S_3 + \ddot{\theta}_2 C_3$

2. (a)-(b) Results given in problem definition.