

Intermediate Dynamics

Exercises #8b Answers

1. Lagrangian:

$$L = \left(\frac{1}{2}m_P \ell^2 S_\theta^2\right) \dot{\theta}^2 + \left(\frac{1}{6}m \ell^2\right) \dot{\theta}^2 - m_P g \ell C_\theta - \frac{1}{2}m g \ell C_\theta - \frac{1}{2}k \ell^2 S_\theta^2$$

Differential Equation of Motion:

$$\left[\frac{1}{3}m \ell^2 + m_P \ell^2 S_\theta^2\right] \ddot{\theta} + (m_P \ell^2 S_\theta C_\theta) \dot{\theta}^2 + (c \ell^2 C_\theta^2) \dot{\theta} + k \ell^2 S_\theta C_\theta - [m_P g + \frac{1}{2}m g + F(t)] \ell S_\theta = 0$$

2. Lagrangian:

$$L = \frac{1}{2}(m_1 + m_2) \dot{x}^2 + \left(\frac{1}{6}m_2 \ell^2\right) \dot{\theta}^2 + \left(\frac{1}{2}m_2 \ell C_\theta\right) \dot{x} \dot{\theta} - \frac{1}{2}k x^2 + \frac{1}{2}m_2 g \ell C_\theta$$

Differential Equations of Motion:

$$(m_1 + m_2) \ddot{x} + \left(\frac{1}{2}m_2 \ell C_\theta\right) \ddot{\theta} - \left(\frac{1}{2}m_2 \ell S_\theta\right) \dot{\theta}^2 + c \dot{x} + k x = F(t)$$

$$\left(\frac{1}{3}m_2 \ell^2\right) \ddot{\theta} + \left(\frac{1}{2}m_2 \ell C_\theta\right) \ddot{x} + \frac{1}{2}m_2 g \ell S_\theta = 0$$

3. Kinetic Energy:

$$K = \frac{1}{4}m_S r^2 \dot{\phi}^2 + \frac{1}{24}m \ell^2 (\dot{\theta}^2 + \dot{\phi}^2 S_\theta^2)$$

Differential Equations of Motion:

$$\left[\frac{1}{2}m_S r^2 + \frac{1}{12}m \ell^2 S_\theta^2\right] \ddot{\phi} + \left(\frac{1}{6}m \ell^2 S_\theta C_\theta\right) \dot{\theta} \dot{\phi} = M_\phi$$

$$\left(\frac{1}{12}m \ell^2\right) \ddot{\theta} - \left(\frac{1}{12}m \ell^2 S_\theta C_\theta\right) \dot{\phi}^2 = M_\theta$$

4. Lagrangian:

$$L = \left[\frac{1}{4}m_d R^2 + \frac{1}{2}m \left(b + \frac{1}{2} \ell S_\theta\right)^2 + \frac{1}{24}m \ell^2 S_\theta^2\right] \dot{\phi}^2 + \left(\frac{1}{6}m \ell^2\right) \dot{\theta}^2 - \frac{1}{2}k \theta^2 + \frac{1}{2}m g \ell C_\theta$$

Differential Equations of Motion:

$$\left[\frac{1}{2}m_d R^2 + m \left(b + \frac{1}{2} \ell S_\theta\right)^2 + \frac{1}{12}m \ell^2 S_\theta^2\right] \ddot{\phi} + \left[m b \ell C_\theta + \frac{2}{3}m \ell^2 S_\theta C_\theta\right] \dot{\theta} \dot{\phi} = M_\phi$$

$$\left(\frac{1}{3}m \ell^2\right) \ddot{\theta} - \left[\frac{1}{2}m b \ell C_\theta + \frac{1}{3}m \ell^2 S_\theta C_\theta\right] \dot{\phi}^2 + c \dot{\theta} + k \theta + \frac{1}{2}m g \ell S_\theta = M_\theta$$