Intermediate Dynamics
Exercises #10 Answers

1. $\theta_{eq} = 0$ and $\theta_{eq} = 45$ (deg)

2. $$\begin{bmatrix} (m_1 + m_2) & \frac{1}{2} m_2 \ell \\ \frac{1}{2} m_2 \ell & \frac{1}{3} m_2 \ell^2 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} c & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} k & 0 \\ 0 & \frac{1}{2} m_2 g \ell \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} F(t) \\ 0 \end{Bmatrix}$$

   $$\{\omega_n\} \approx \begin{Bmatrix} 4.89 \\ 29.44 \end{Bmatrix} \text{(r/s)} \approx \begin{Bmatrix} 0.7784 \\ 4.686 \end{Bmatrix} \text{(Hz)}$$

   Mode 1: $\underset{\sim}{u}_1 \approx \begin{Bmatrix} 1 \\ 77.8 \end{Bmatrix}$ Mode 2: $\underset{\sim}{u}_2 \approx \begin{Bmatrix} 1 \\ -0.7715 \end{Bmatrix}$

3. $\theta_{eq} = 0$ and $\theta_{eq} = 90^0$

   $\theta_{eq} = 0 \quad \Rightarrow \quad \Delta\ddot{\theta} - \Omega^2(\Delta\theta) = 0$ (unstable equilibrium)

   $\theta_{eq} = 90^0 \quad \Rightarrow \quad \Delta\ddot{\theta} + \Omega^2(\Delta\theta) = 0$ (stable equilibrium)

   $M_\phi = \frac{1}{6} m \ell^2 S_\theta C_\theta \Omega \dot{\theta}$

4. $\theta_{eq} = 58.7^0 \quad \Rightarrow \quad \Delta\ddot{\theta} + (133.52)\Delta\theta = 0$ (stable equilibrium)

   $M_\phi = \frac{2}{3} m \ell^2 S_\theta C_\theta \Omega \dot{\theta}$