Multibody Dynamics
Representation of Vector Operations as Matrix Operations

Dot (or Scalar) Product

Given two vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ both expressed in terms of the unit vector set $\left(\underset{\sim}{e}_1, \underset{\sim}{e}_2, \underset{\sim}{e}_3\right)$

$$\underset{\sim}{a} = a_1\,\underset{\sim}{e}_1 + a_2\,\underset{\sim}{e}_2 + a_3\,\underset{\sim}{e}_3$$
$$\underset{\sim}{b} = b_1\,\underset{\sim}{e}_1 + b_2\,\underset{\sim}{e}_2 + b_3\,\underset{\sim}{e}_3$$

then the dot (or ***scalar***) product of the two vectors is defined to be

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = \sum_{i=1}^{3} a_i b_i \,.$$

This product can be represented as the following ***matrix product***.

$$\underset{\sim}{a} \cdot \underset{\sim}{b} \quad \rightarrow \quad \{a\}^T \{b\}$$

Here, $\{a\}$ indicates a 3×1 ***column vector*** whose elements are the components of the vector $\underset{\sim}{a}$, and $\{a\}^T$ indicates a 1×3 ***row vector*** that is the ***transpose*** of $\{a\}$. Recall for any two vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$, $\underset{\sim}{a}\cdot\underset{\sim}{b} = \underset{\sim}{b}\cdot\underset{\sim}{a}$. That is, the dot product is ***commutative***. The matrix equivalent of this statement is $\{a\}^T \{b\} = \{b\}^T \{a\} = \left(\{a\}^T \{b\}\right)^T$.

Cross (or Vector) Product

Again, given two vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ both expressed in terms of the same unit vector set

$$\underset{\sim}{a} = a_1\,\underset{\sim}{e}_1 + a_2\,\underset{\sim}{e}_2 + a_3\,\underset{\sim}{e}_3$$
$$\underset{\sim}{b} = b_1\,\underset{\sim}{e}_1 + b_2\,\underset{\sim}{e}_2 + b_3\,\underset{\sim}{e}_3$$

then the cross (or ***vector***) product of the two vectors is defined to be

$$\underset{\sim}{a}\times\underset{\sim}{b} = (a_2 b_3 - a_3 b_2)\,\underset{\sim}{e}_1 + (a_3 b_1 - a_1 b_3)\,\underset{\sim}{e}_2 + (a_1 b_2 - a_2 b_1)\,\underset{\sim}{e}_3 \,.$$

This product can be represented as the following ***matrix product***

$$\underset{\sim}{a}\times\underset{\sim}{b} \quad \rightarrow \quad [\tilde{a}]\{b\} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix}$$

Here, $[\tilde{a}]$ is a ***skew-symmetric matrix*** defined (as above) using the components of $\underset{\sim}{a}$. Recall that for any two vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$, $\underset{\sim}{a}\times\underset{\sim}{b} = -\underset{\sim}{b}\times\underset{\sim}{a}$. The matrix equivalent of this statement is $\left[\tilde{a}\right]\{b\} = -\left[\tilde{b}\right]\{a\}$. Recall also that a skew-symmetric matrix is ***equal*** to the ***negative*** of its ***transpose***. So, for example, $\left[\tilde{a}\right]\{b\} = -\left[\tilde{a}\right]^T \{b\}$.