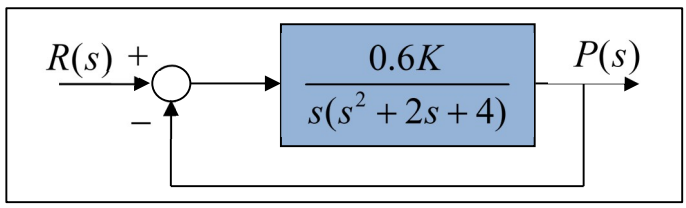
Introductory Motion and Control
What is a Root Locus Diagram?

Definition: A ***root locus diagram*** is the plot of the ***paths*** of the poles of a closed loop system as a single parameter K is varied.

Root Locus Diagram: $0 \le K < +\infty$
Complementary Root Locus Diagram: $-\infty < K \le 0$

Example:

As presented in earlier notes, ***proportional position control*** of a ***space platform*** can be expressed using the block diagram below. The open-loop platform dynamics is ***second-order*** and ***under-damped***.

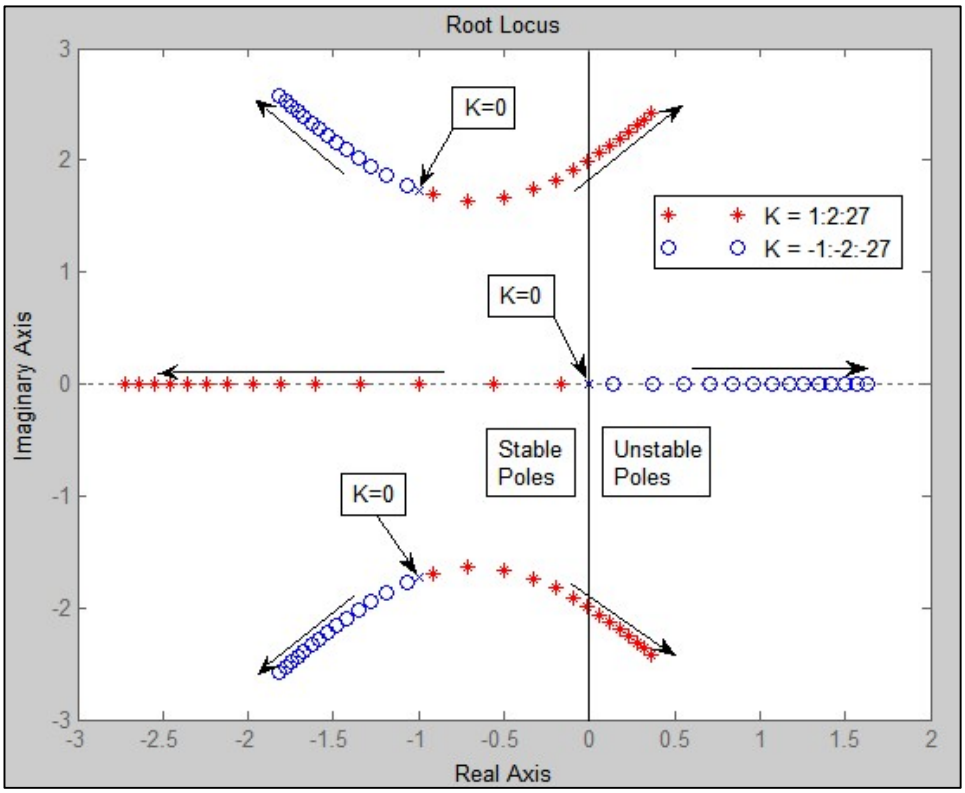


Question: How do the ***poles*** of this closed-loop system ***vary*** as the parameter K changes?

One way to answer this question is simply to find the closed loop transfer function, identify the ***characteristic equation***, and compute the ***roots*** for a variety of parameter values. For this system, the characteristic equation is $\boxed{s^3 + 2s^2 + 4s + 0.6K = 0}$. The figure below shows a ***plot*** of the ***poles*** of the system for a given set of K values. The red stars indicate the locations of the poles for $1 \le K \le 27$ with an increment $\Delta K = 2$. The blue circles indicate the locations of the poles for $-1 \le K \le -27$ with an increment of $\Delta K = -2$.

Observations about this root locus diagram:

- As K ***increases*** (becoming more ***positive***), the ***real pole*** moves from the ***origin*** along the ***negative real axis***, while a pair of ***complex conjugate poles*** crosses over the imaginary axis and moves into the ***right-half plane***. As K ***decreases*** (becoming more negative), the ***real pole*** moves from the origin along the ***positive real axis***, while a pair of ***complex conjugate poles*** moves further into the ***left-half plane***.
- As K increases toward $+\infty$ or decreases toward $-\infty$ the branches follow ***asymptotes***.
- The poles ***do not*** necessarily ***move uniformly*** along the branches.
- Although the poles do not move uniformly along the branches, the branches are ***smooth curves***.
- Recall that if all the poles of a system are in the ***left-half plane***, the system is ***stable***, and if any of the poles are in the ***right-half plane***, it is ***unstable***. This system is stable for $0 \le K \le 13$ (approximately). A ***more precise estimate*** of the upper limit of K can be found using the Routh-Hurwitz method or by specifying smaller increments in MATLAB.


Root Locus
K=0
K = 1:2:27
K = -1:-2:-27
K=0
Stable Poles
Unstable Poles
K=0
Imaginary Axis
Real Axis
3
2
1
0
-1
-2
-3
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2