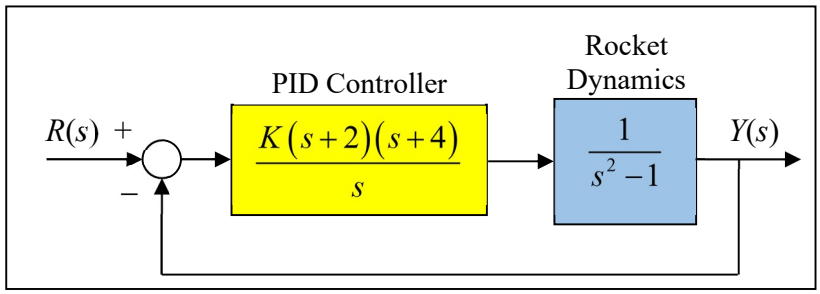
Introductory Motion and Control
Using MATLAB® for Root Locus Analysis

Reference: Dorf & Bishop, Modern Control Systems, Pearson/Prentice-Hall, 11th Ed., 2008.

As an example of how to use MATLAB to perform a root locus analysis, consider design problem **DP**6.4 of Dorf & Bishop. The ***block diagram*** of the closed-loop system is shown below. The goal is to use MATLAB to ***draw*** a ***root locus diagram*** for parameter K.



The ***characteristic equation*** of the closed-loop system is $1+GH(s)=0$ or $1+KP(s)=0$. Substituting the transfer functions from the block diagram into the characteristic equation gives

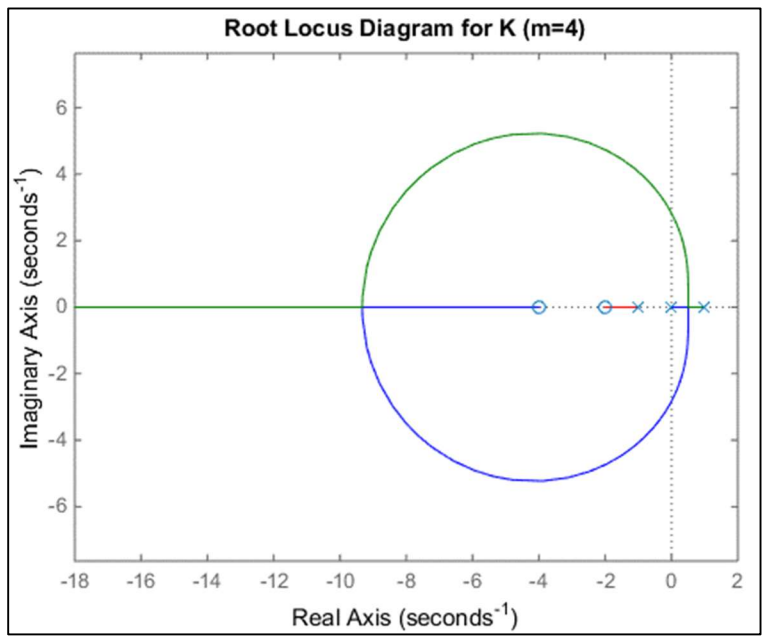
$$\boxed{1+K\left[\frac{(s+4)(s+2)}{s(s^2-1)}\right]=1+K\left[\frac{s^2+6s+8}{s^3-s}\right]=0}$$

The MATLAB commands that produce the root locus diagram are:

```
num = [1,6,8];
den = [1, 0, -1, 0];

sys = tf(num,den)

rlocus(sys)
axis ('equal')
title('Root Locus Diagram for K (m=4)')
```



In response to the "`tf`" command, MATLAB provides the ***transfer function*** in the ***command window***.

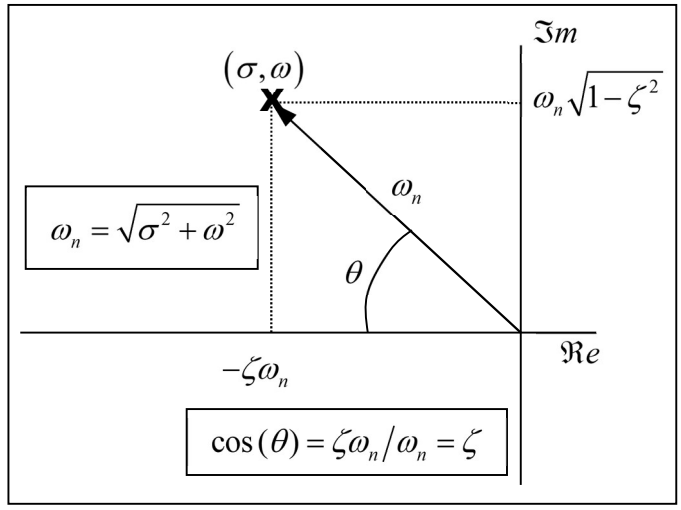
```
sys =
  s^2 + 6 s + 8
  -------------
     s^3 - s
Continuous-time transfer function.
```

The ***three branches*** of the diagram are shown in green, blue, and red. Note, however, that MATLAB ***does not show*** the ***direction*** of the ***movement*** of the poles as K increases. It is ***understood*** the movement is ***from the poles*** of $P(s)$ ***to the zeros*** of $P(s)$. In this case, there are ***three branches***, but only ***two zeros*** so one of the branches

must go to infinity along an asymptote. As stated in previous notes, if there is only one asymptote, it is at 180 degrees. The "`axis ('equal')`" command ensures that the diagram is shown in its ***true shape***.

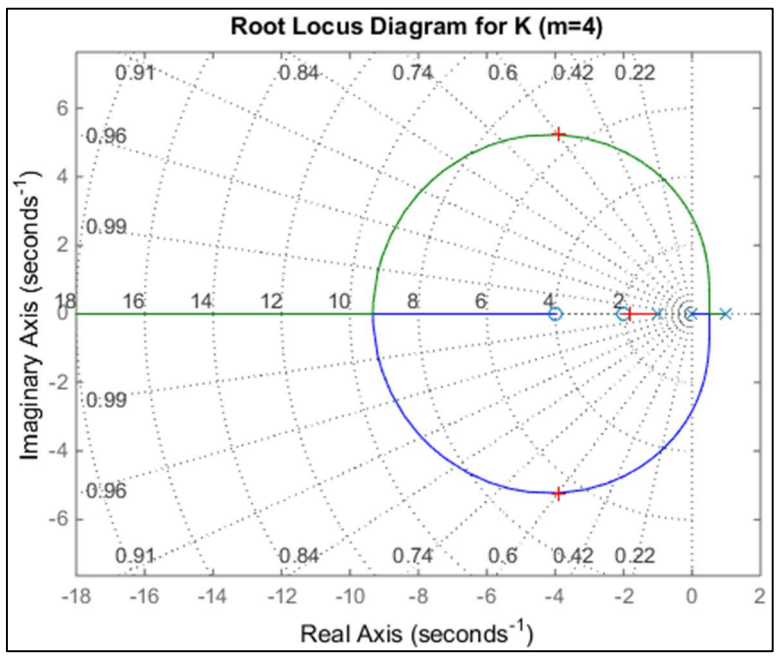
Target Regions for Poles

- The ***damping ratios*** and ***settling times*** of the poles are determined by their ***location*** in the *s*-plane.
- To ensure a settling time ***less than*** T_s, the ***real parts*** of all the poles of the system should be to the ***left*** of $4/T_s$.
- The ***damping ratio*** of each of the complex poles is determined by drawing a vector from the origin to the location of the pole and measuring the angle θ between this vector and the negative real axis.
- The damping ratio is calculated as $\zeta = \cos(\theta)$. For example, poles that lie below the $\theta = 45^{\circ}$ line have damping ratios $\zeta > 0.707$.



Parameter Values Associated with Poles in the Target Region

To find parameter values associated with poles within the target region, use the "`rlocfind`" command in MATLAB. After executing the "`rlocfind`" command, click on a desirable pole location on one of the branches of the root locus diagram in the plot window. MATLAB automatically picks the point on the branch that is closest to your selection.



The MATLAB commands are:

```
grid
[k, poles] = rlocfind(sys)
```

Note the grid is not the usual rectangular grid but rather a ***polar grid*** which aligns with ***constant values*** of ***damping ratio*** (radial lines) and ***frequency*** (concentric circles). In response to the user clicking on the selected point, MATLAB places a "+" at the location of the associated poles, and the following data is provided in the command window.

```
selected point = -3.900473933649288 + 5.249262688362950i

k = 9.609310443185597

poles =

 -3.901567707687948 + 5.228747819083528i
 -3.901567707687948 - 5.228747819083528i
 -1.806175027809703 + 0.000000000000000i
```

So, in this case, for a K value of approximately 9.61, the closed-loop system has ***one real pole*** at -1.8 and a ***pair of complex conjugate poles*** at $-3.90 \pm 5.23j$. The complex poles have a damping ratio of approximately $\zeta = 0.6$. The frequency associated with the real pole is 1.8 (rad/s), and the frequency associated with the complex poles is 6.52 (rad/s). The settling time associated with the real pole is $4/1.8 \approx 2.22$ (sec), and the settling time associated with the complex poles is $4/3.90 \approx 1.03$ (sec).