# Multibody Dynamics
# Angular Acceleration Using Absolute Coordinates

## Angle Derivatives as Generalized Speeds: 1-2-3 Rotation Sequence

Recall that the fixed-frame and body-frame components of ${}^{R}\underset{\sim}{\omega}_B$ the angular velocity of a body can be written in matrix-vector form as follows.

$$\boxed{\begin{Bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{Bmatrix}=\begin{bmatrix}1 & 0 & S_2\\ 0 & C_1 & -S_1C_2\\ 0 & S_1 & C_1C_2\end{bmatrix}\begin{Bmatrix}\dot\theta_1\\ \dot\theta_2\\ \dot\theta_3\end{Bmatrix}=\left[\omega_{B,\dot\theta}\right]\left\{\dot\theta\right\}} \qquad \text{"Fixed-frame Components"} \qquad (1)$$

$$\boxed{\begin{Bmatrix}\omega'_1\\ \omega'_2\\ \omega'_3\end{Bmatrix}=\begin{bmatrix}C_2C_3 & S_3 & 0\\ -C_2S_3 & C_3 & 0\\ S_2 & 0 & 1\end{bmatrix}\begin{Bmatrix}\dot\theta_1\\ \dot\theta_2\\ \dot\theta_3\end{Bmatrix}=\left[\omega'_{B,\dot\theta}\right]\left\{\dot\theta\right\}} \qquad \text{"Body-frame Components"} \qquad (2)$$

Consequently, the fixed-frame and body-frame components of ${}^{R}\underset{\sim}{\alpha}_B$ the angular acceleration of the body can be written in the following matrix-vector forms. Recall that the angular velocity vector can be differentiated in either the fixed-frame or the body-frame to find the angular acceleration.

$$\boxed{\begin{Bmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{Bmatrix}=\begin{Bmatrix}\dot\omega_1\\ \dot\omega_2\\ \dot\omega_3\end{Bmatrix}=\left[\omega_{B,\dot\theta}\right]\left\{\ddot\theta\right\}+\left[\dot\omega_{B,\dot\theta}\right]\left\{\dot\theta\right\}} \qquad \text{"Fixed-frame Components"} \qquad (3)$$

$$\boxed{\begin{Bmatrix}\alpha'_1\\ \alpha'_2\\ \alpha'_3\end{Bmatrix}=\begin{Bmatrix}\dot\omega'_1\\ \dot\omega'_2\\ \dot\omega'_3\end{Bmatrix}=\left[\omega'_{B,\dot\theta}\right]\left\{\ddot\theta\right\}+\left[\dot\omega'_{B,\dot\theta}\right]\left\{\dot\theta\right\}} \qquad \text{"Body-frame Components"} \qquad (4)$$

The time derivatives of the partial angular velocity matrices are

$$\left[\dot\omega_{B,\dot\theta}\right]=\begin{bmatrix}0 & 0 & \dot\theta_2C_2\\ 0 & -\dot\theta_1S_1 & \left(S_1S_2\dot\theta_2-C_1C_2\dot\theta_1\right)\\ 0 & \dot\theta_1C_1 & -\left(S_1C_2\dot\theta_1+C_1S_2\dot\theta_2\right)\end{bmatrix} \qquad \left[\dot\omega'_{B,\dot\theta}\right]=\begin{bmatrix}-\left(S_2C_3\dot\theta_2+C_2S_3\dot\theta_3\right) & C_3\dot\theta_3 & 0\\ \left(S_2S_3\dot\theta_2-C_2C_3\dot\theta_3\right) & -S_3\dot\theta_3 & 0\\ C_2\dot\theta_2 & 0 & 0\end{bmatrix}$$

## Angular Velocity Components as Generalized Speeds

If the ***angular velocity components*** are used as ***generalized speeds***, then we have a much simpler form of the angular acceleration. Recalling that the partial velocity matrices associated with the angular velocity components are $3\times3$ identity matrices,

$$\boxed{\begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{Bmatrix} = \left[\omega_{B,\omega}\right]\left\{\dot{\omega}\right\} + \left[\dot{\omega}_{B,\omega}\right]\left\{\omega\right\} = \left[\omega_{B,\omega}\right]\left\{\dot{\omega}\right\} = \left\{\dot{\omega}\right\}}$$

"Fixed-frame Components"

$$\boxed{\begin{Bmatrix} \alpha'_1 \\ \alpha'_2 \\ \alpha'_3 \end{Bmatrix} = \left[\omega'_{B,\omega'}\right]\left\{\dot{\omega}'\right\} + \left[\dot{\omega}'_{B,\omega'}\right]\left\{\omega'\right\} = \left[\omega'_{B,\omega'}\right]\left\{\dot{\omega}'\right\} = \left\{\dot{\omega}'\right\}}$$

"Body-frame Components"