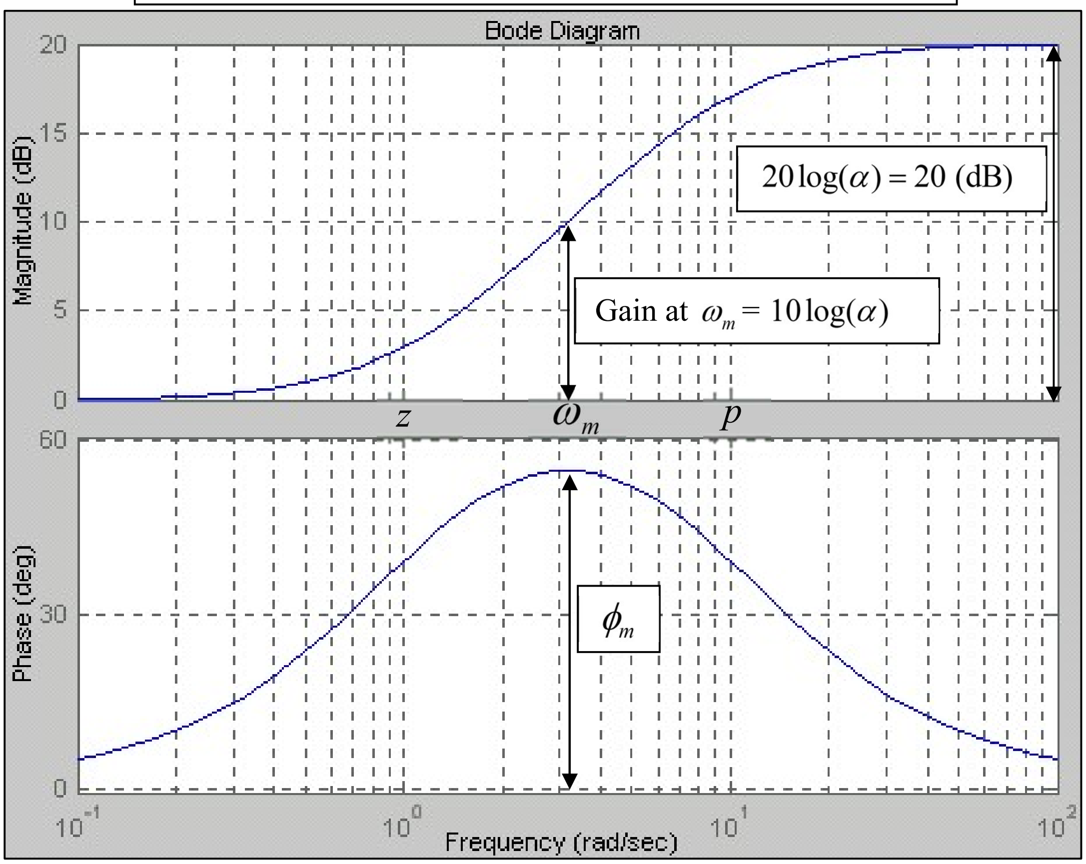
# Introductory Motion and Control
# Phase-Lead Compensator Design

Reference: Dorf & Bishop, Modern Control Systems, 10th Ed, Prentice-Hall, 2005.

**Phase-Lead Compensator** (PD type compensator)

- Transfer function of a ***phase-lead*** compensator: $G_c(s) = \alpha\left(\dfrac{s+z}{s+p}\right)$, where $|z| < |p|$
- The multiplier $\alpha = |p|/|z|$
- As a result, the compensator ***amplifies*** at frequencies near and above the location of the zero and ***increases*** the phase of the system near the ***pole*** and ***zero*** locations, as well.
- A Bode diagram of the phase-lead compensator $G_c(s) = 10\left(\dfrac{s+1}{s+10}\right)$ is shown below.

Bode Diagram of the Example Phase-Lead Compensator




- Here, $\alpha = 10$, the ***logarithmic mean frequency*** is $\omega_m = \sqrt{|pz|} = 3.16$ (rad/s), the overall magnitude increase is $20\log(\alpha) = 20$ (dB), and the phase shift at the mean frequency is $\phi_m = \sin^{-1}\left(\dfrac{\alpha-1}{\alpha+1}\right) = +54.9$ (deg).

**Phase-Lead Compensator Design Using Bode Diagrams**

- ***Find loop gain*** $K$ required to satisfy the ***steady-state error requirement*** (if given).
- Evaluate the ***phase margin (PM)*** of the ***uncompensated system*** with the loop gain $K$ to determine if proportional control is sufficient.
- ***Find*** $\phi_a$ the ***necessary additional phase*** required to give the desired phase margin.
- ***Find*** $\alpha$ for the ***compensator*** using the equation $\alpha = \dfrac{1+\sin(\phi_a)}{1-\sin(\phi_a)}$.
- Examine the ***Bode plot*** of the ***uncompensated system*** (with the loop gain $K$) to find the frequency where $M = -10\log(\alpha)$. Define this frequency to be $\omega_m$ the ***logarithmic mean frequency*** of the compensator. This will be the new ***zero*-dB *crossover*** point.
- ***Find*** the ***pole*** and ***zero*** locations $p = \omega_m\sqrt{\alpha}$ and $z = p/\alpha$, and define the compensator to be $G_c(s) = K\alpha\left(\dfrac{s+z}{s+p}\right)$.
- ***Check*** the ***phase margin*** of the ***compensated system*** to see if the desired value has been attained. If not, then decide on the additional phase required, and repeat the steps above starting with the calculation of $\alpha$.
- ***Simulate*** the time-domain performance.

**Phase-Lead Compensator Design Using Root Locus Diagrams**

- ***Target Regions*: Set *damping ratio*** $(\zeta)$ and ***natural frequency*** $(\omega_n)$ values for the complex poles (assuming they are all dominant and have no influence from transfer function zeros) to target a ***desirable percent overshoot*** and ***settling time***.
- ***Examine*** the ***uncompensated root locus diagram*** to see if the pole locations determined above can be met with only proportional control.
- ***Add*** a ***zero*** and ***pole*** to $GH(s)$ to move root locus branches into the target region. Use these additions to alter the shape of the root locus diagram in predictable ways. Experience helps.
  - Can change the locations of real poles of the closed loop system
  - Separation between the pole and zero will ***move all asymptotes to the left***.
  - The location of any ***closed-loop zeros*** may cause overshoot problems.
- ***Simulate*** the time-domain performance.