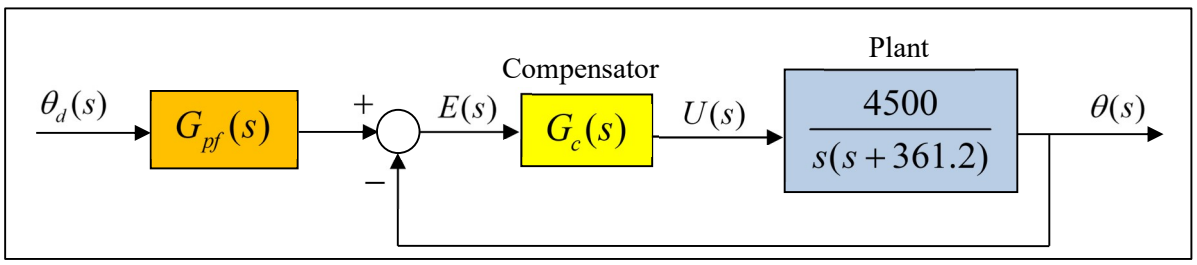
Introductory Motion and Control
Control Effort

One major concern of control system designers is: "***Can the physical system generate the control effort required to give the desired performance?***" To check this, the designer must understand not only the total closed-loop system response, but also the ***response of individual components*** of the system and how those components interact.

Consider the ***aircraft attitude control system*** shown in the block diagram below. To ensure proper performance of the plant (the aircraft), the control signal $U(s)$ (output of the compensator) must be in the appropriate range for proper plant operation. If it is ***too large***, the "real" plant may not provide the proper response. In this case, if the wing flaps are commanded to too large an angle, then the flap will "stall" and not produce the forces necessary to control the aircraft.



<u>Example</u>: Control Effort of a PID Compensator

In previous notes, a PID compensator and pre-filter were designed for this system. The transfer functions of the compensator and pre-filter are of the following form.

$$\boxed{G_c(s) = \frac{K(s^2 + as + b)}{s}} \qquad \text{and} \qquad \boxed{G_{pf}(s) = \frac{(1143)^3}{1639(s^2 + as + b)}}$$

For a good step response, the parameters were chosen to have the following values: $K = 0.36423$, $a = 1713.73$, and $b = 9.1107 \times 10^5$. Recall that parameter a is the ratio of the proportional and derivative gains, and parameter b is the ratio of the integral and derivative gains.

To find the control signal $U(s)$ associated with a step input, the transfer function $U(s)/\theta_d(s)$ is found for systems with and without a pre-filter. Using block diagram reduction,

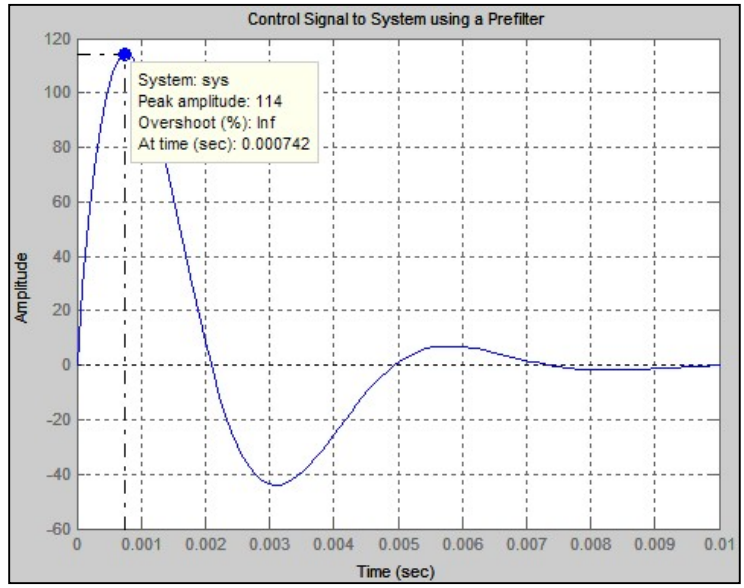
$$\boxed{\frac{U(s)}{\theta_d(s)} = \frac{(3.31845 \times 10^5)\, s\,(s + 361.2)}{s^3 + 2000s^2 + (2.8089 \times 10^6)s + 1.49327 \times 10^9}} \qquad \text{With Pre-Filter} \qquad (1)$$

$$\boxed{\frac{U(s)}{\theta_d(s)} = \frac{K\, s\,(s + 361.2)\,(s^2 + as + b)}{s^3 + 2000s^2 + (2.8089 \times 10^6)s + 1.49327 \times 10^9}} \qquad \text{Without Pre-Filter} \qquad (2)$$

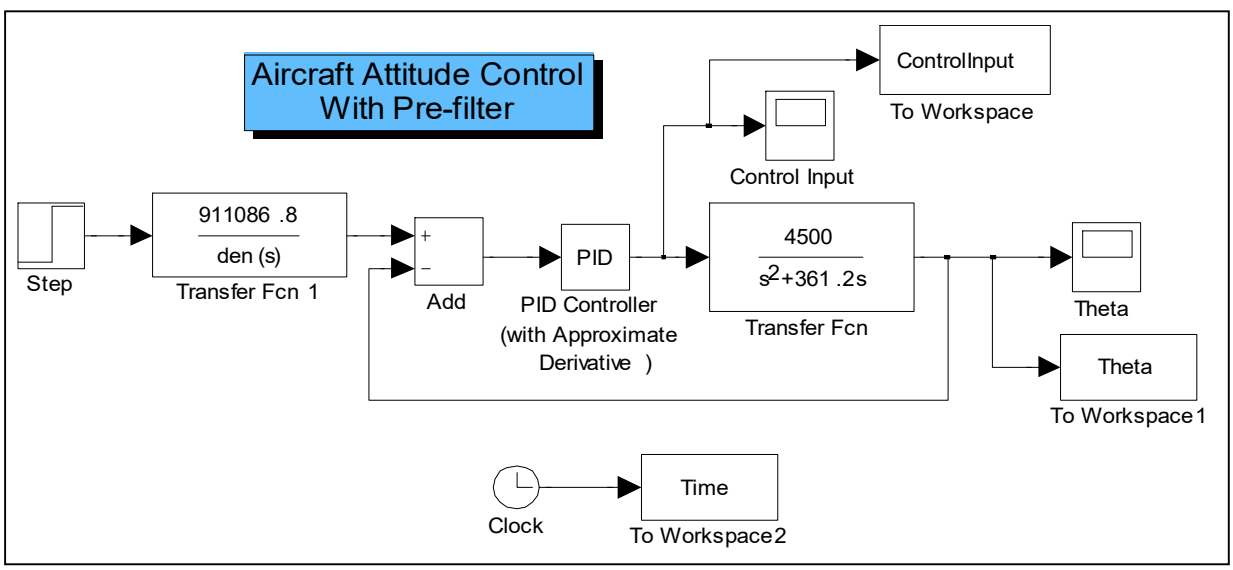
Note that the transfer function for the system ***with a pre-filter*** is ***proper***, while the transfer function for the system ***without a pre-filter*** is ***not proper***. The step command in MATLAB cannot be used with improper transfer functions. It is shown below how to use Simulink to find the control signal for the system without a pre-filter.

Control Signal with a Pre-Filter

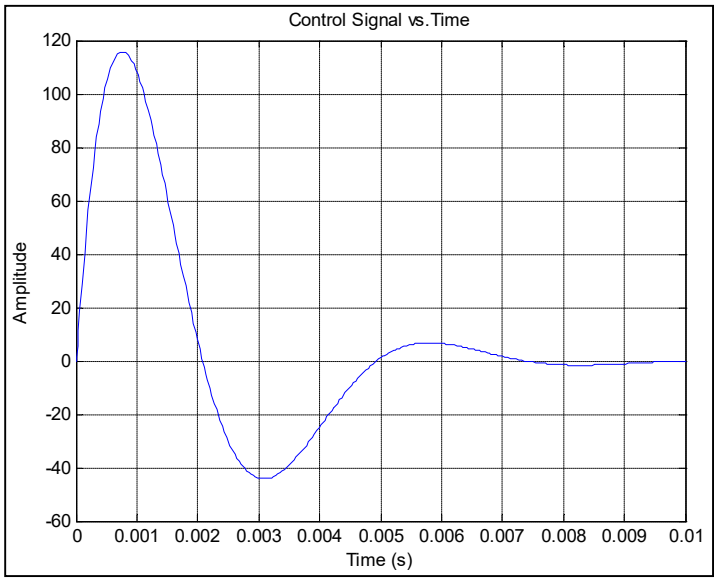
Assuming a step input for the desired attitude angle $\theta_d(s)$, the control signal for the system with a pre-filter can be found in various ways. The figure to the right shows the signal found ***using the transfer function*** of Eq. (1) and MATLAB's step command. The system designer must now decide if that signal is within the range of acceptable inputs for the aircraft.

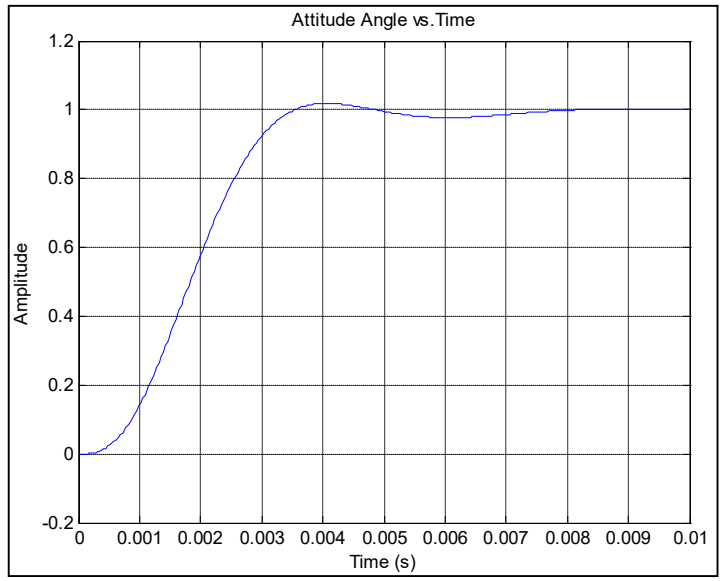


The ***control signal*** can also be found using the following Simulink model.



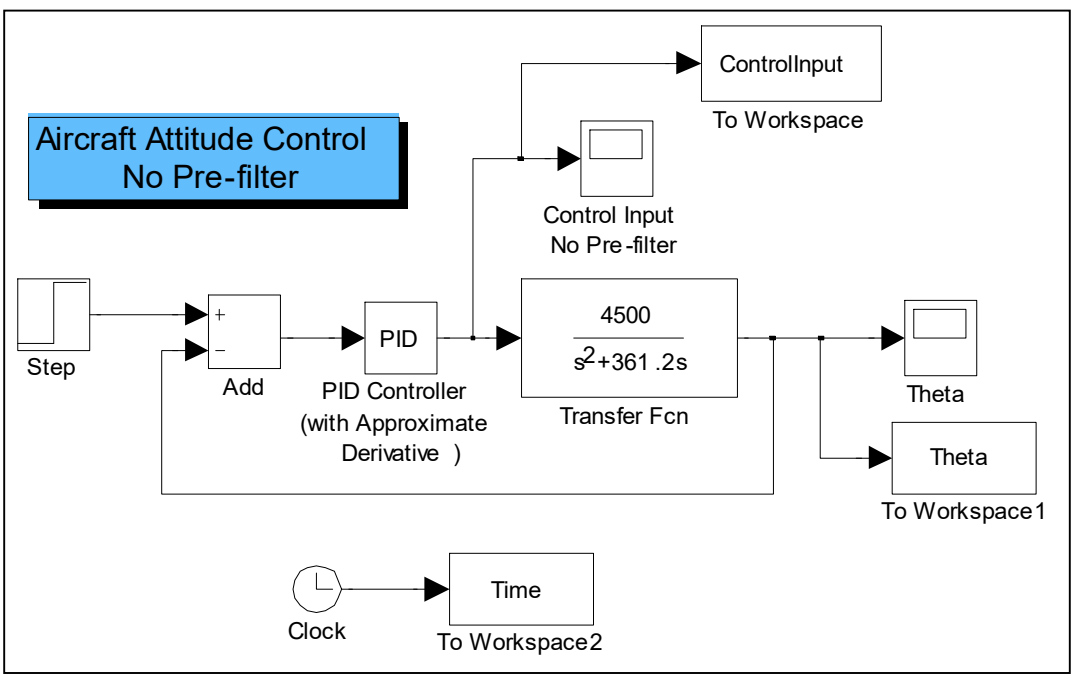
Using a ***Simulink model***, the designer has access to all a system's signals. The above model records the ***time signals*** for the ***control input*** and the ***attitude angle***. These two signals are plotted below for a unit step input. Note the control signal is the same as the one (presented above) calculated using MATLAB's step function. The attitude angle signal is the same as that presented in the previous notes entitled "Compensator Design for ITAE Optimal Response with Pre-Filters".



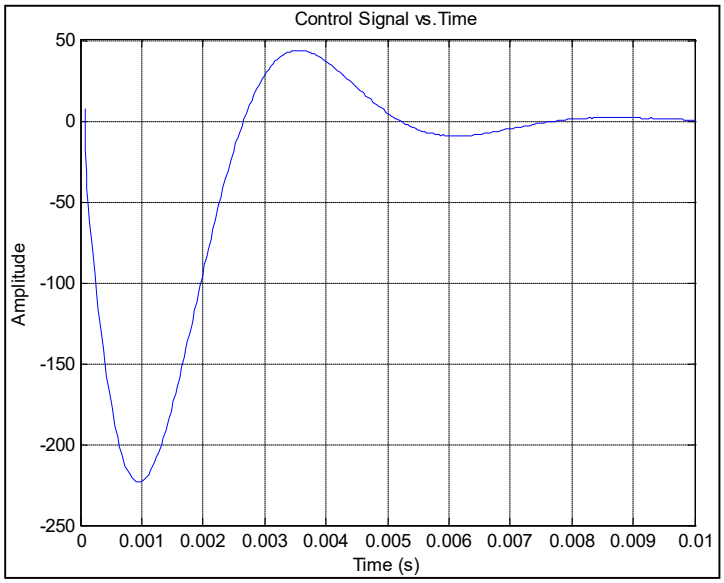


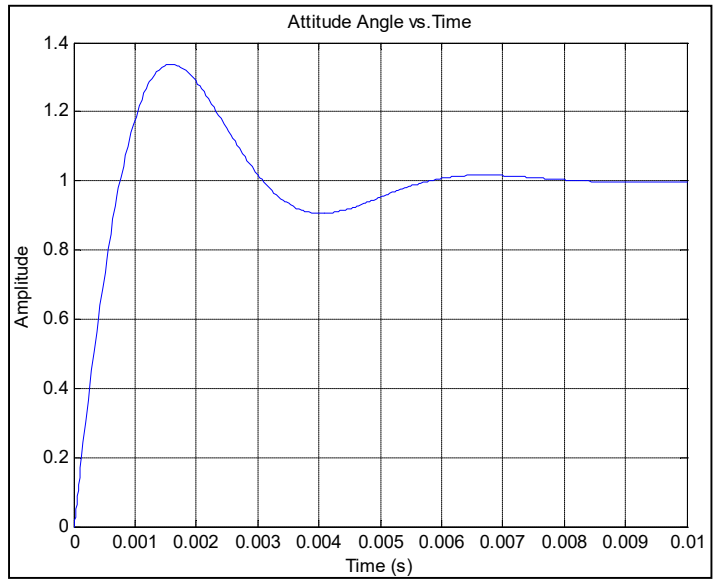
Control Signal without a Pre-Filter

As noted above, the control signal transfer function for the system without a pre-filter is ***not a proper transfer function***. In that case, it is convenient to use Simulink to estimate the control signal. The diagram below shows the Simulink model of the system ***without pre-filter***.



As the previous model, this model records the ***time signals*** for the ***control input*** and the ***attitude angle***. These signals are plotted below for a unit step input. Note there is a ***discontinuity*** in the ***control signal*** at $t = 0$ due to the derivative control term, so the first few points are omitted from the plot. Note also that the plot of the attitude angle signal is identical to that shown in previous notes entitled "Compensator Design for ITAE Optimal Response with Pre-Filters".





Summary

The ***control signal*** in the system ***without*** the ***pre-filter*** has a large initial spike and the maximum value that follows at $t \approx 0.001$ (sec) is almost twice as large as the maximum value for the control signal for the system with the pre-filter. By "***softening***" the input to the control loop, the pre-filter has ***reduced*** the load on the actuation system (e.g. flaps on the aircraft). The higher loads required by the system without the pre-filter may not even be physically possible.

Another way of looking at pre-filters is that they can be used to generate a ***command input signal*** (or ***command trajectory***) that is easier for the system to follow. As a direct result of this feature, it ***reduces*** the load on the actuation system. If properly designed, pre-filters can be used to ***avoid saturation*** of the actuation system.