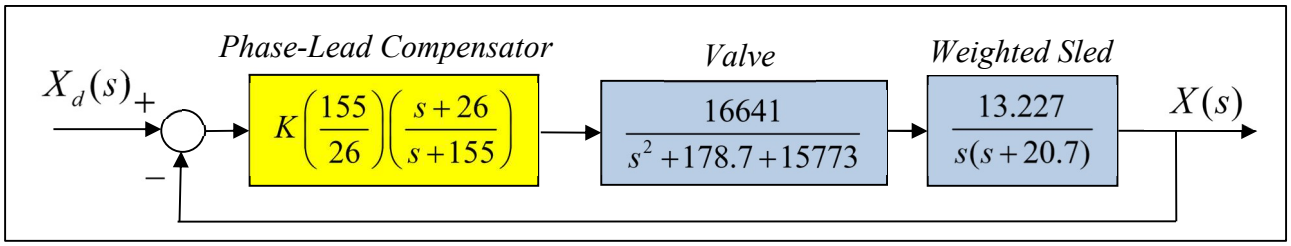
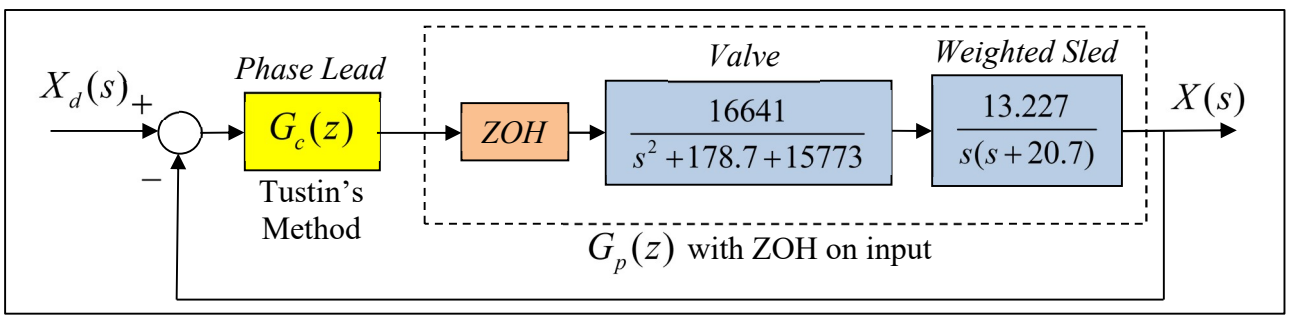
Introductory Motion and Control
Discrete Root Locus Analysis of a Weighted Sled System

Consider ***position control*** of a weighted sled system using a ***continuous*** phase-lead compensator as shown in the diagram below.



Closed-Loop Position Control of a Weighted Sled System

To determine the ***effects*** of ***discretization*** on the stability and ***performance*** of this system, a ***discrete root locus analysis*** can be done. ***Discrete equivalents*** for the ***valve*** and ***weighted sled*** transfer functions are found by assuming there is a ***zero-order hold*** on the valve input. Tustin's method is used here to find a discrete equivalent for the phase-lead compensator.



Discrete Closed-Loop Position Control of a Weighted Sled System

Using the "***c2d***" function in MATLAB and a ***sample time*** of $T = 0.01$ (sec), the following results are obtained for the ***compensator*** and ***valve/sled*** transfer functions.

```
z = 26; p = 155;
numGc = p*[1,z]; denGc = z*[1,p]; sysGc = tf(numGc,denGc);

sysGcD = c2d(sysGc,T,'tustin')

   Transfer function:
      3.795 z - 2.922
      ---------------
      z - 0.1268

   Sampling time: 0.01
```

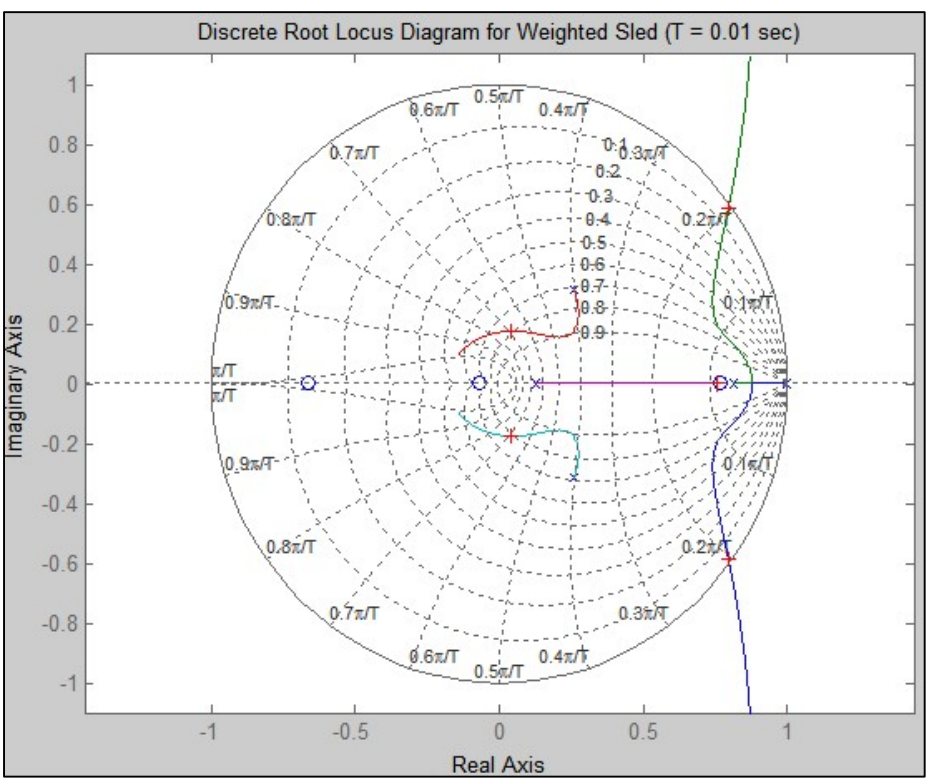
```
numGv= 16641.6; denGv= [1,178.688,15772.5]; sysGv= tf(numGv,denGv)

numGs = 13.227; denGs = [1,20.7,0]; sysGs = tf(numGs,denGs)

sysGp = series(sysGv,sysGs); sysGpD = c2d(sysGp,T,'zoh')

  Transfer function:
     6.103e-005 z^3 + 0.0004431 z^2 + 0.000294 z + 1.832e-005
     ---------------------------------------------------------
      z^4 - 2.333 z^3 + 1.923 z^2 - 0.7263 z + 0.1362

  Sampling time: 0.01
```

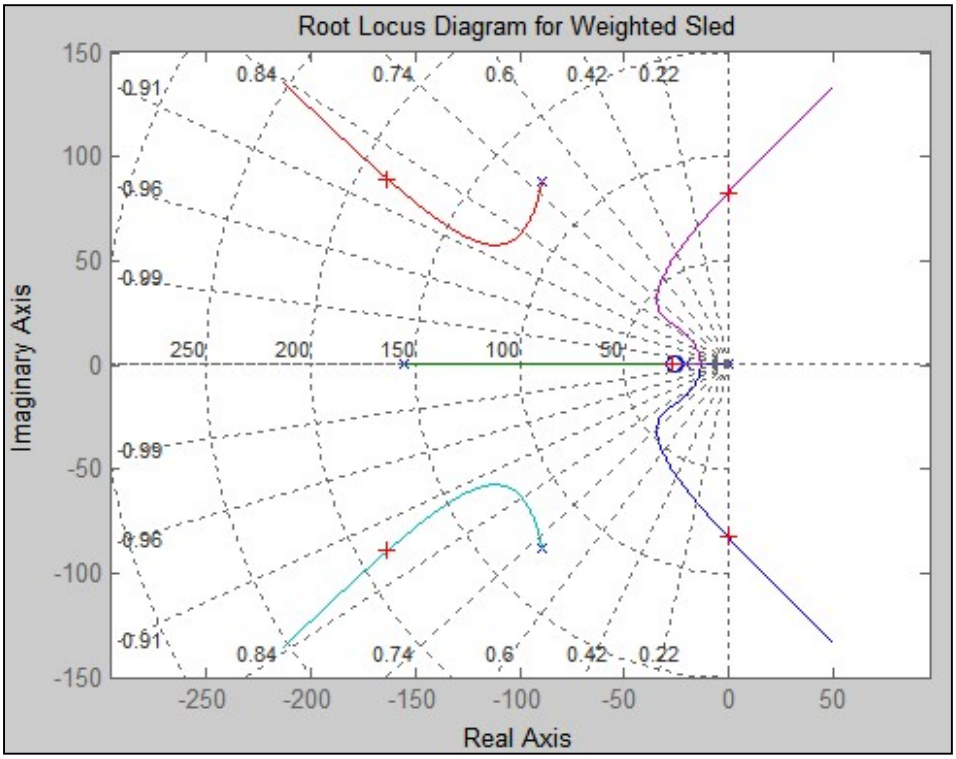
Given $G_c(z)$ and $G_p(z)$ the discretized transfer functions of the compensator and the valve/sled plant, the ***discrete closed loop transfer function*** can be computed.

$$\boxed{\frac{X}{X_d}(z) = \frac{G_c(z)G_p(z)}{1+G_c(z)G_p(z)}}$$

As with continuous systems, the ***characteristic equation*** of the closed loop system with negative unity feedback is $\boxed{1+G_c(z)G_p(z)=0}$.

Using the "***rlocus***" and "***rlocfind***" commands, the ***limiting gain*** for stability is $K \approx 125$.



Note that the ***stability boundary*** is now the unit circle.

Contrast these results to the root locus diagram for the continuous system shown below. Here the ***limiting gain*** is found to be $K \approx 187$.