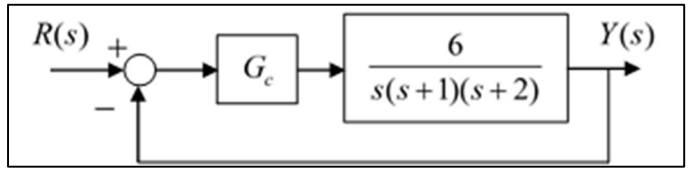
Introductory Motion and Control
Exercises #5 Phase-lag Compensator Design Using Bode Diagrams

1. The block diagram shows a third order system to be controlled by a *phase-lag* compensator $G_c(s)$.

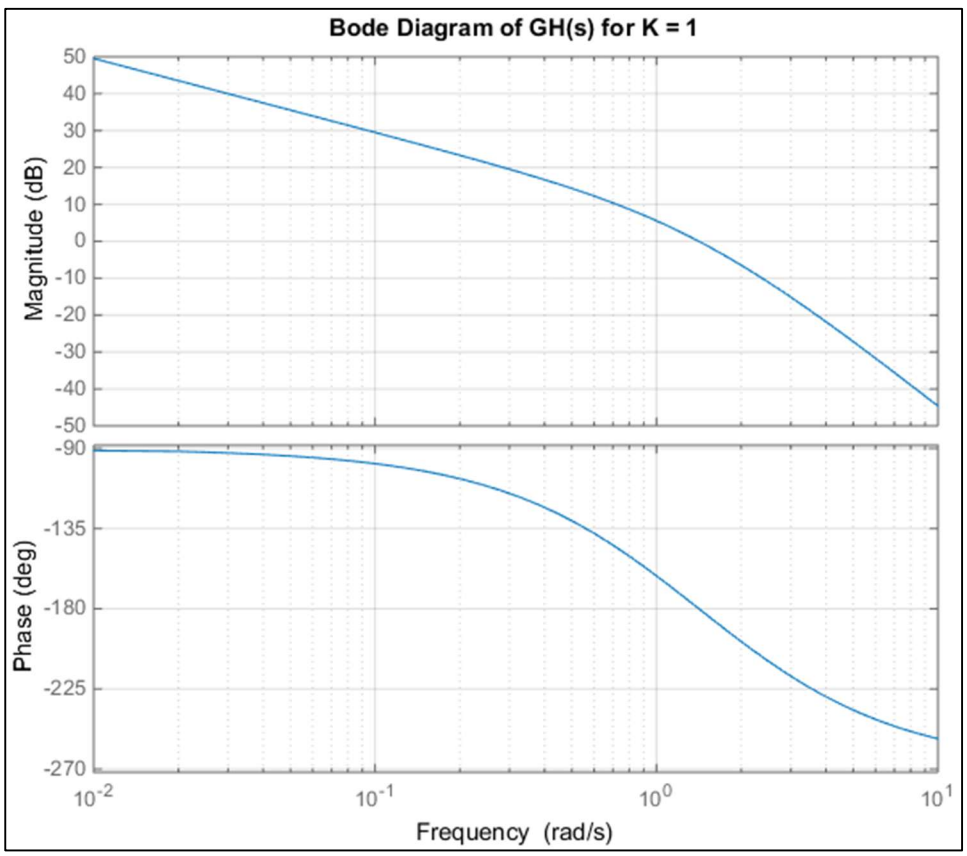


a) Assuming $G_c(s) = K$, find the range of K values required to make e_{ss} the steady state error due to a unit ramp input less than 0.1. b) Design a phase lag compensator that will allow the closed-loop system to satisfy the error specification of part (a) and have a phase margin of $PM = 40$ (deg). The Bode diagram for $GH(s)$ with $K = 1$ is shown below.

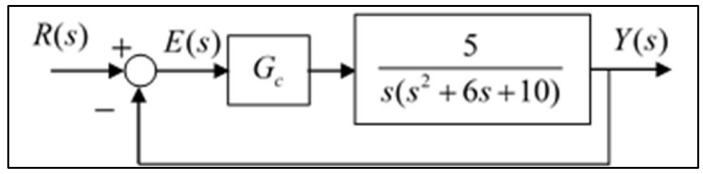
Answers:

a) $K > 3.33$. Using $K = 6$, the current system is unstable. b) $G_c(s) = 6 \times 0.03981 \times \left[\dfrac{s+0.06}{s+0.00239}\right]$

Note: The compensated system has a phase margin of approximately 37.2 (deg). More iteration is required to meet the phase margin requirement.



2. The block diagram shows a third order system to be controlled by a *phase-lag* compensator $G_c(s)$.

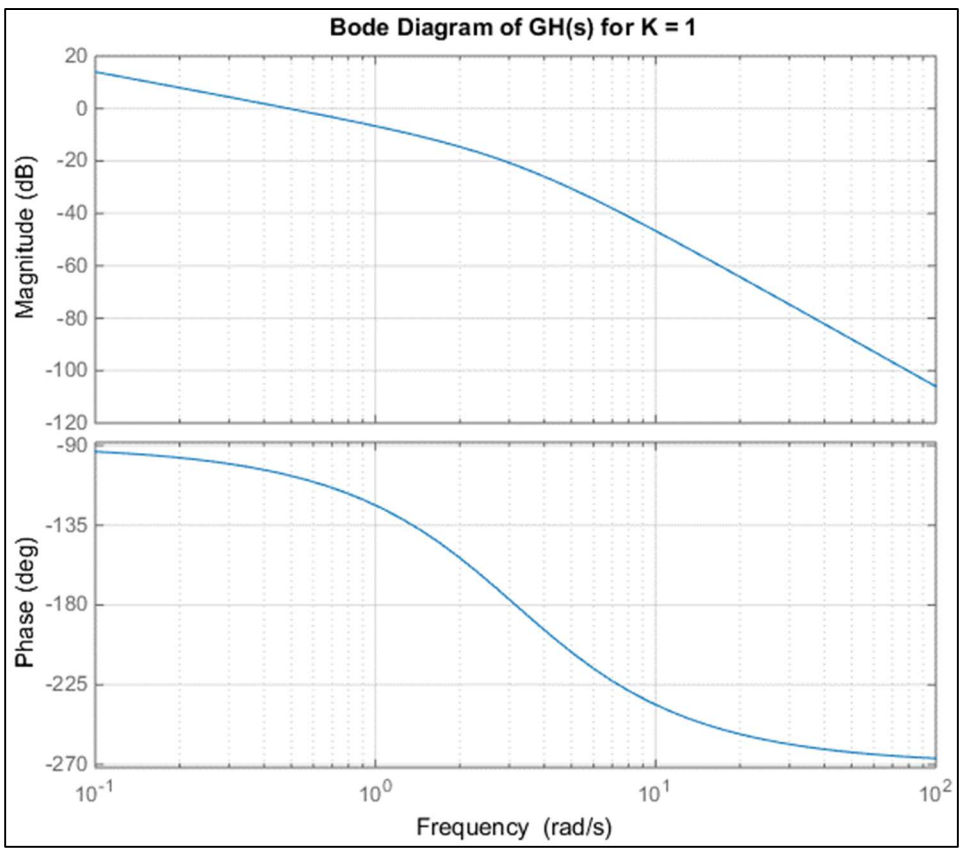


a) Assuming $G_c(s) = K$, find the range of K values required to make e_{ss} the steady state error due to a unit ramp input less than 0.1. b) Design a phase lag compensator that will allow the closed-loop system to satisfy the error specification of part (a) and have a phase margin of $PM = 60$ (deg). The Bode diagram for $GH(s)$ with $K = 1$ is shown below.

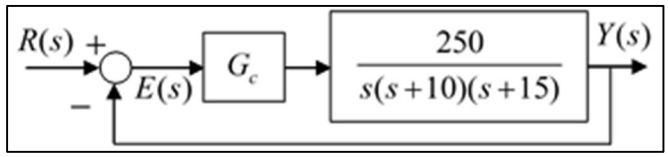
Answers:

a) $K \geq 20$. Using $K = 20$, the current system is unstable. b) $G_c(s) = 20 \times 0.1 \times \left[\dfrac{s+0.08}{s+0.008}\right]$

Note: The compensated system has a phase margin of approximately 54 (deg). More iteration is required to meet the phase margin requirement.



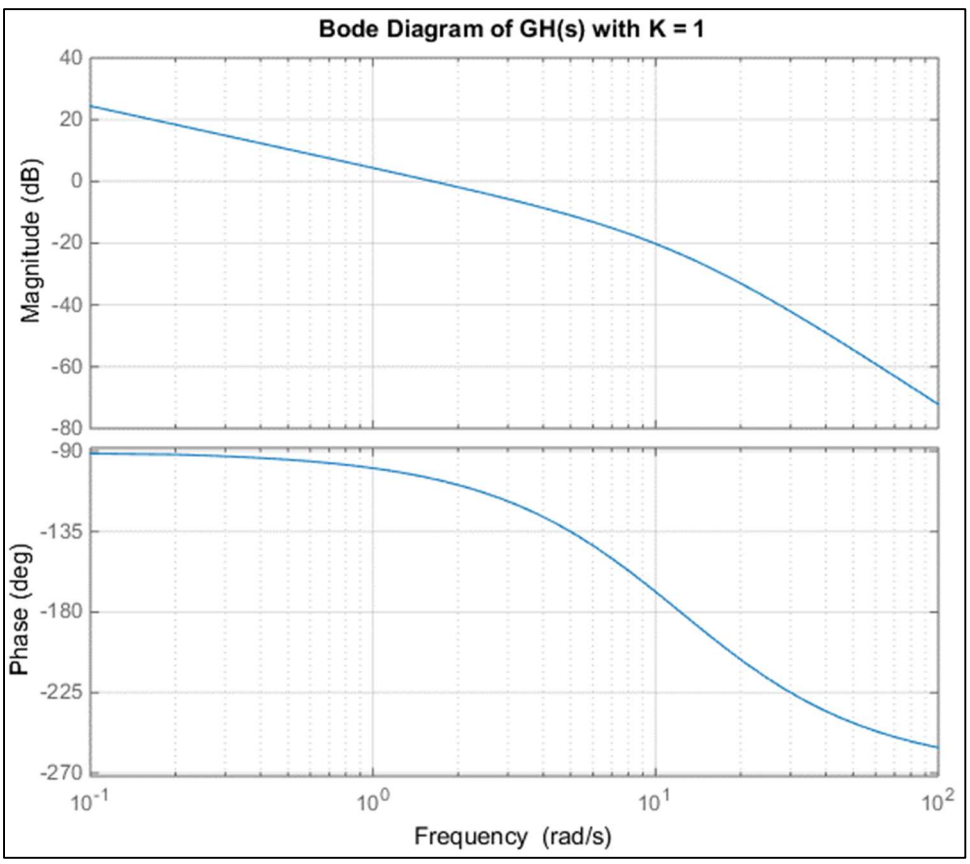
3. The block diagram shows a third order system to be controlled by a *phase-lag* compensator $G_c(s)$.



a) Assuming $G_c(s) = K$, find the range of K values required to make e_{ss} the steady state error due to a unit ramp input less than or equal to 0.06. b) Design a phase lag compensator that will allow the closed-loop system to satisfy the error specification of part (a) and have a phase margin of $PM = 45$ (deg). The Bode diagram for $GH(s)$ with $K = 1$ is shown below.

Answers:

a) $K \geq 10$. Using $K = 10$, the system has a phase margin of approximately 11 degrees.

b) $G_c(s) = 10 \times 0.3162 \times \left[\dfrac{s+0.5}{s+0.1581}\right]$

Note: The compensated system has a phase margin of approximately 44 (deg). The result is very close to the desired margin.