Multibody Dynamics
Exercises #2 Answers

a) $\left\{\omega'_{B_i}\right\} = \left[\omega'_{B_i,\dot{\theta}_i}\right]\left\{\dot{\theta}_i\right\} \quad (i=1,2) \qquad \left\{\theta_i\right\} \triangleq \left[\theta_{i1} \quad \theta_{i2} \quad \theta_{i3}\right]^T \quad \left(i=1,2\right)$ (the angles for body "i")

$$\left[\omega'_{B_i,\dot{\theta}_i}\right] = \begin{bmatrix} -S_{i2} & 0 & 1 \\ C_{i2}S_{i3} & C_{i3} & 0 \\ C_{i2}C_{i3} & -S_{i3} & 0 \end{bmatrix} \quad (i=1,2)$$

$$\left[\omega'_{B_i,\dot{\theta}_i}\right] = \left[0\right]_{3\times 3} \quad (i \neq j)$$

b) $\left\{p_{G_1}\right\} = \left\{s_1\right\} + \left[R_{B_1}\right]^T \left\{r'_1\right\}$

$$\left\{p_{G_2}\right\} = \left\{s_1\right\} + \left[R_{B_1}\right]^T \left(\left\{q'_2\right\} + \left\{s'_2\right\}\right) + \left[R_{B_2}\right]^T \left\{r'_2\right\}$$

$$\left[R_{B_i}\right] = \begin{bmatrix} C_{i1}C_{i2} & S_{i1}C_{i2} & -S_{i2} \\ -S_{i1}C_{i3} + C_{i1}S_{i2}S_{i3} & C_{i1}C_{i3} + S_{i1}S_{i2}S_{i3} & C_{i2}S_{i3} \\ S_{i1}S_{i3} + C_{i1}S_{i2}C_{i3} & -C_{i1}S_{i3} + S_{i1}S_{i2}C_{i3} & C_{i2}C_{i3} \end{bmatrix}$$

c) $\left\{v_{G_i}\right\} = \left[v_{G_i,\dot{s}_1}\right]\left\{\dot{s}_1\right\} + \left[v_{G_i,\dot{\theta}_1}\right]\left\{\dot{\theta}_1\right\} + \left[v_{G_i,\dot{s}'_2}\right]\left\{\dot{s}'_2\right\} + \left[v_{G_i,\dot{\theta}_2}\right]\left\{\dot{\theta}_2\right\} \quad (i=1,2)$

Here,

$$\left[v_{G_1,\dot{s}_1}\right] = \left[v_{G_2,\dot{s}_1}\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[v_{G_1,\dot{s}'_2}\right] = \left[0\right]_{3\times 3}$$

$$\left[v_{G_1,\dot{\theta}_1}\right] = -\left[R_{B_1}\right]^T \left[\tilde{r}'_1\right]\left[\omega'_{B_1,\dot{\theta}_1}\right]$$

$$\left[v_{G_1,\dot{\theta}_{B_2}}\right] = \left[0\right]_{3\times 3}$$

$$\left[v_{G_2,\dot{s}'_2}\right] = \left[R_{B_1}\right]^T$$

$$\left[v_{G_2,\dot{\theta}_1}\right] = -\left[R_{B_1}\right]^T \left(\left[\tilde{q}'_2\right] + \left[\tilde{s}'_2\right]\right)\left[\omega'_{B_1,\dot{\theta}_1}\right]$$

$$\left[v_{G_2,\dot{\theta}_2}\right] = -\left[R_{B_2}\right]^T \left[\tilde{r}'_2\right]\left[\omega'_{B_2,\dot{\theta}_2}\right]$$