Multibody Dynamics
Exercises #7 Answers (d'Alembert's Principle)

1. $\left[\frac{1}{3}m\ell^2 + m_P\ell^2 S_\theta^2\right]\ddot{\theta} + (m_P\ell^2 S_\theta C_\theta)\dot{\theta}^2 + (c\ell^2 C_\theta^2)\dot{\theta} + k\ell^2 S_\theta C_\theta - \left[m_P g + \frac{1}{2}mg + F(t)\right]\ell S_\theta = 0$

2. $(m_1 + m_2)\ddot{x} + (\frac{1}{2}m_2\ell C_\theta)\ddot{\theta} - (\frac{1}{2}m_2\ell S_\theta)\dot{\theta}^2 + c\dot{x} + kx = F(t)$

$$(\tfrac{1}{3}m_2\ell^2)\ddot{\theta} + (\tfrac{1}{2}m_2\ell C_\theta)\ddot{x} + \tfrac{1}{2}m_2 g\ell S_\theta = 0$$

3. $\left[\frac{1}{2}m_S r^2 + \frac{1}{12}m\ell^2 S_\theta^2\right]\ddot{\phi} + (\frac{1}{6}m\ell^2 S_\theta C_\theta)\dot{\theta}\dot{\phi} = M_\phi$

$$(\tfrac{1}{12}m\ell^2)\ddot{\theta} - (\tfrac{1}{12}m\ell^2 S_\theta C_\theta)\dot{\phi}^2 = M_\theta$$

4. $\left[\frac{1}{2}m_d R^2 + m(b + \frac{1}{2}\ell S_\theta)^2 + \frac{1}{12}m\ell^2 S_\theta^2\right]\ddot{\phi} + (mb\ell C_\theta + \frac{2}{3}m\ell^2 S_\theta C_\theta)\dot{\theta}\dot{\phi} = M_\phi$

$$(\tfrac{1}{3}m\ell^2)\ddot{\theta} - \left[\tfrac{1}{2}mb\ell C_\theta + \tfrac{1}{3}m\ell^2 S_\theta C_\theta\right]\dot{\phi}^2 + c\dot{\theta} + k\theta + \tfrac{1}{2}mg\ell S_\theta = M_\theta$$