Multibody Dynamics
Exercises #9 Answers

1. There are ***four*** equations for $\left(\phi,\theta,\omega_1',\omega_2'\right)$:

$$\boxed{\begin{gathered}\left(m\ell^2/12\right)S_\theta\dot{\omega}_1'+\left(m\ell^2/12\right)C_\theta\omega_1'\omega_2'=-M_\phi\\ \left(m\ell^2/12\right)\dot{\omega}_2'-\left(m\ell^2C_\theta/12S_\theta\right)\omega_1'^2=M_\theta\end{gathered}}$$

with the kinematical differential equations

$$\boxed{\begin{aligned}&\dot{\phi}=-\omega_1'/S_\theta\\&\dot{\theta}=\omega_2'\end{aligned}}$$

2. There are ***seven*** equations for $\left(\omega_1',\omega_2',\omega_3',\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4\right)$:

$$\boxed{\begin{aligned}\left(I+mL^2\right)\dot{\omega}_1'+\left(I_3-I-mL^2\right)\omega_2'\omega_3'&=2mgL\left(\varepsilon_2\varepsilon_3+\varepsilon_1\varepsilon_4\right)\\ \left(I+mL^2\right)\dot{\omega}_2'+\left(I+mL^2-I_3\right)\omega_1'\omega_3'&=2mgL\left(\varepsilon_2\varepsilon_4-\varepsilon_1\varepsilon_3\right)\\ \omega_3'&=\text{ constant}\end{aligned}}$$

with the kinematical differential equations

$$\boxed{\left\{\dot{\varepsilon}\right\}_{4\times1}=\frac{1}{2}\begin{bmatrix}\varepsilon_4&-\varepsilon_3&\varepsilon_2&\varepsilon_1\\ \varepsilon_3&\varepsilon_4&-\varepsilon_1&\varepsilon_2\\ -\varepsilon_2&\varepsilon_1&\varepsilon_4&\varepsilon_3\\ -\varepsilon_1&-\varepsilon_2&-\varepsilon_3&\varepsilon_4\end{bmatrix}\begin{Bmatrix}\omega_1'\\ \omega_2'\\ \omega_3'\\ 0\end{Bmatrix}}$$

3. a) Partial velocities and partial angular velocities

$$\begin{array}{lll}\partial\underset{\sim}{v}_G/\partial\omega_1'=L\underset{\sim}{e}_3/2 & \partial\underset{\sim}{v}_G/\partial\omega_2'=0 & \partial\underset{\sim}{v}_G/\partial\omega_3'=-L\underset{\sim}{e}_1/2\\ \partial\underset{\sim}{v}_A/\partial\omega_1'=L\underset{\sim}{e}_3 & \partial\underset{\sim}{v}_A/\partial\omega_2'=0 & \partial\underset{\sim}{v}_A/\partial\omega_3'=-L\underset{\sim}{e}_1\\ \partial\underset{\sim}{v}_B/\partial\omega_1'=L\underset{\sim}{e}_3/2 & \partial\underset{\sim}{v}_B/\partial\omega_2'=-L\underset{\sim}{e}_3/2 & \partial\underset{\sim}{v}_B/\partial\omega_3'=(L/2)(-\underset{\sim}{e}_1+\underset{\sim}{e}_2)\end{array}$$

b) Generalized forces

$$\boxed{F_{\omega_1'}=LF_{A3}+LF_B/2+mgLS_1C_2}\quad\boxed{F_{\omega_2'}=-LF_B/2}\quad\boxed{F_{\omega_3'}=-LF_{A1}+mgL(C_1S_3+S_1S_2C_3)}$$

c) Equations of motion

$$\boxed{\begin{aligned}\left(\frac{7}{12}mL^2\right)\dot{\omega}_1'+\left(\frac{7}{12}mL^2\right)\omega_2'\omega_3'&=F_{\omega_1'}\\ \left(\frac{1}{12}mL^2\right)\dot{\omega}_2'-\left(\frac{1}{12}mL^2\right)\omega_1'\omega_3'&=F_{\omega_2'}\\ \left(\frac{2}{3}mL^2\right)\dot{\omega}_3'-\left(\frac{1}{2}mL^2\right)\omega_1'\omega_2'&=F_{\omega_3'}\end{aligned}}$$

d) Kinematical differential equations

$$\boxed{\begin{aligned}\dot{\theta}_1&=\left(\omega_1'C_3-\omega_2'S_3\right)/C_2\\ \dot{\theta}_2&=\omega_1'S_3+\omega_2'C_3\\ \dot{\theta}_3&=\omega_3'+\left(-\omega_1'C_3+\omega_2'S_3\right)S_2/C_2\end{aligned}}$$