

An Introduction to Three-Dimensional, Rigid Body Dynamics

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Volume III – Introduction to Multibody Kinematics

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Unit 1 – Background Material

This unit introduces the use of *matrix methods* to perform the basic mathematical operations required for the development of the equations of motion for multibody systems. This allows the equations to be expressed in a more *compact form* that is easily converted into *computer algorithms*. These algorithms can be used to *reliably* generate the equations of motion of complex systems rather than trying to generate them by hand. It should be stressed that a *thorough understanding* of the equations in this form depends heavily on the *analyst's grasp* of the *vector-based methods* presented in Volumes I and II.

This unit also reviews the concept of *generalized coordinates*, *generalized speeds*, *state vectors*, and *body-connection* and *ending-body* arrays.

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Unit 2 – Angular Velocity and Partial Angular Velocity

This unit focuses on the *matrix-based* calculation of *vector* components of *angular velocity* and *partial angular velocity matrices*. The calculations are performed using *fixed frame* and *body frame* components and are based on *absolute* and *relative coordinates*. Both *orientation angle derivatives* and *angular velocity* components are used as *generalized speeds*. Algorithms are developed for the efficient calculation of these quantities for multibody systems.

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Unit 3 – Angular Acceleration

This unit focuses on the *matrix-based* calculation of *vector* components of *angular acceleration*. The calculations are performed using *fixed frame* and *body frame* components and are based on *absolute* and *relative coordinates*. Both *orientation angle derivatives* and *angular velocity components* are used as *generalized speeds*. As part of the calculations, a procedure for calculating the *time derivatives* of *relative transformation matrices* (between moving bodies) is also included.

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Unit 4 – Velocity and Partial Velocity

This unit focuses on the *matrix-based* calculation of components of *velocity vectors* and *partial velocity matrices*. The calculations are based on *absolute* and *relative coordinates*. Both *orientation angle derivatives* and *angular velocity components* can be used as *generalized speeds*. Algorithms are developed for the efficient calculation of these quantities for multibody systems.

Explicit results are generated for some *examples* with the *purpose* of being *clear* about *how* the calculations are done. However, keep in mind that the *goal* of developing such procedures is to *implement* them into *computer algorithms*.

Results for calculating the *derivatives* of *transformation matrices* presented in Units 1 and 3 of this volume are *repeated* in this unit for convenience of the reader.

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Unit 5 – Acceleration

This unit focuses on *matrix-based* calculations of *fixed frame components of acceleration*. The accelerations are expressed in terms of partial velocity matrices and their time derivatives. Both *base frame* and *body frame* components of *relative angular velocity vectors* are used in the calculations. Both *absolute* and *relative coordinates* are discussed. Both *orientation angle derivatives* and *angular velocity components* can be used as *generalized speeds*.

Explicit results are generated for some *examples* with the *purpose* of being *clear* about *how* the calculations are done. However, keep in mind that the *goal* of developing such procedures is to *implement* them into *computer algorithms*.

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References:

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